

## MATH 115 – Linear Algebra for Engineers Fall 2010

**Objectives:** MATH 115 is an introduction to abstract vector spaces, the algebra and geometry of  $\mathbf{R}^n$ , and the abstract theory of vector spaces. We approach the material through a blend of theoretical ideas and computational methods, such as Gaussian elimination for solving systems of linear equations. Various applications are also studied.

**Text:** Linear Algebra with Applications, 6<sup>th</sup> Edition, Keith Nicholson.

**Course Home Page:** Information about the course will be posted on the course web page, on UW-ACE

### Summary:

1. Review of Vectors Chapter 4, Sections 4.1 – 4.2
2. Linear Equations Chapter 1, Sections 1.1 – 1.6
2. Matrix Algebra Chapter 2, Sections 2.1 – 2.5
3. Determinants Chapter 3, Sections 3.1 – 3.3
4. Linear operators Chapter 4, Section 4.4.
5. Real Vector Spaces Chapter 5, Sections 5.1 – 5.5
6. Orthogonality Chapter 8, Sections 8.1, 8.2
7. Abstract Vector Spaces Chapter 6, Sections 6.1 – 6.4
8. Complex Numbers Appendix A

**Final Marks:**

Labs	15%
Midterm test	20%
Final Exam	65%

Calculators are not permitted on the midterm or on the Final. **Midterm date:** Thursday October 21, 7 to 9 pm (to be determined)

**Tutorials and Labs:** In addition to 3 lecture hours, there is a two-hour tutorial scheduled each week. Each week, a 2 hour Lab assignment will be handed out.

- The lab **will normally contain questions related to the material presented in the week preceding the lab**. For example the lab given in week 6 covers material presented in week 5. Each Friday the instructor will specify which material will be covered on the lab either in class or via the Ace webpage.
- Students must complete this lab assignment during the Lab period. You may work in groups, and ask the TA for help, but you must hand in your own assignment.
- *The best way to prepare for these labs* is to work out, **before the lab**, all the assigned exercises listed below. They are meant to enhance understanding and to help you absorb the material. Mathematics is a skill developed by working out problems.

**Note for students with disabilities:**

The Office for Persons with Disabilities (OPD), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the OPD at the beginning of each academic term.

**Students are expected to know what constitutes academic integrity, to avoid committing academic offences, and to take responsibility for their actions.**

Students who are unsure whether an action constitutes an offence, or who need help in learning how to avoid offences (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, TA, academic advisor, or the Undergraduate Associate Dean. For information on categories of offences and types of penalties, students should refer to Policy #71, Student Academic Discipline . Students who believe that they have been wrongfully or unjustly penalized have the right to grieve; refer to Policy#70, Student Grievance.

**Homework Problems:** You will find the recommended homework problems listed in Schedule table below. Although these problems will not be handed in for grading, it is strongly recommended that students work through these problems carefully as they will help prepare for the weekly labs and will form the basis for the midterm and the final exam.

## Recommended schedule

Week	Dates	Sections and topics	Lab	Assigned exercises
1	Sept. 13-17 (Lectures 1 to 3)	<b>Section 4.1</b> : Vectors and lines <ul style="list-style-type: none"> <li>• Vectors in <math>\mathbb{R}^2</math> and <math>\mathbb{R}^3</math>, dot products</li> <li>• vectors as directed line segments</li> <li>• Lines in <math>\mathbb{R}^2</math> and <math>\mathbb{R}^3</math></li> </ul>	No lab	<b>§4.1</b> : #1, 4, 7, 14, 21, 22, 24 a), c)
		<b>Section 4.2</b> : Projections and planes <ul style="list-style-type: none"> <li>• Planes in <math>\mathbb{R}^3</math> (<b>Omit cross-products</b>)</li> </ul>		<b>§4.2</b> : 1, 3, 8, 11, 12, 21, 25, 39, 44
2	Sept 20-24 (Lectures 4 to 7)  <b>(4 lectures this week For most sections because of make-up lecture)</b>	<b>Section 4.2</b> : Projections and planes <ul style="list-style-type: none"> <li>• Projections of vectors to lines in <math>\mathbb{R}^3</math></li> </ul>	1	<b>§4.2</b> : 1, 3, 8, 11, 12, 21, 25, 39, 44
		<b>Sections 1.1, 1.2</b> : Systems of linear equations <ul style="list-style-type: none"> <li>• Gauss-Jordan elimination</li> </ul>		<b>§1.1</b> : 1, 2 <b>§1.2</b> : 1, 2, 5, 9, 11, 12, 13, 16, 21
		<b>Section 1.3</b> : Homogeneous systems <ul style="list-style-type: none"> <li>• Linear combination of solutions</li> </ul>		<b>§1.3</b> : 1, 2, 7, 9
		<b>Section 2.1</b> : Matrix algebra <ul style="list-style-type: none"> <li>• Addition, scalar, transposition</li> </ul>		<b>§2.1</b> : 2, 3, 6, 12, 13, 20
3	Sept. 27-Oct 1 (Lectures 8 to 10)	<b>Section 2.2</b> : Matrix equations, matrix transformations <ul style="list-style-type: none"> <li>• Matrix-vector multiplication</li> <li>• Matrix-vector mult. viewed as a transformation</li> </ul>	2	<b>§2.2</b> : 1, 2, 4 a) b), 6, 7, 11, 12, 13
		<b>Section 2.3</b> : Matrix multiplication. <ul style="list-style-type: none"> <li>• Matrix multiplication and block-multiplication (<b>Omit Directed graphs</b>)</li> </ul>		<b>§2.3</b> : 2, 6, 21, 36, 27, 36 Block matrices: 11, 12, 13
		<b>Section 2.4</b> : Matrix inverse <ul style="list-style-type: none"> <li>• Properties and characterizations of invertible matrices</li> <li>• Matrix inversion algorithm</li> <li>• Inverse of a matrix transformation</li> </ul>		<b>§2.4</b> : 1, 2, 3, 4, 9, 16, 22, 28, 30
		<b>Section 2.5</b> : Elementary matrices <ul style="list-style-type: none"> <li>• Inverses and elementary matrices (<b>Omit Smith Normal forms</b>)</li> </ul>		<b>§2.5</b> : 1, 2, 3 a), 7 a), 8 b), 9

4	Oct. 4-8 (Lectures 11 to 14)  (4 lectures this week For most sections because of make-up lecture)	<b>Section 2.6</b> : Matrix transformations <ul style="list-style-type: none"> <li>Linear Transformations</li> </ul>	3	§2.6 : 1, 3, 7, 8 a), 9, 17
		<b>Section 3.1</b> Determinants <ul style="list-style-type: none"> <li>Cofactor expansion</li> <li>Properties of determinants</li> <li>Determinants of triangular matrices</li> <li>Determinants of certain block matrices</li> </ul>		§3.1 : 2, 4, 5 a), 6, 7, 10, 17
		<b>Section 3.2</b> Determinants and matrix inverses <ul style="list-style-type: none"> <li>Determinants of products, inverses, transposes</li> <li>Cramer's rule</li> <li>Polynomial interpolation</li> </ul>		§3.2 : 2, 3, 8, 9 a), 12, 16, 23, 24, a)
5	Oct.12-15 (Lectures 15, 16)  Mon. Oct. 11 is Thanksgiving.  (For most sections, only two lectures this week)	<b>Section 3.2</b> Determinants and matrix inverses <ul style="list-style-type: none"> <li>Determinants of products, inverses, transposes</li> <li>Cramer's rule</li> <li>Polynomial interpolation</li> </ul>	4	§3.2 : 2, 3, 8, 9 a), 12, 16, 23, 24, a)
		<b>Section 3.3</b> Diagonalization and eigenvalues. <ul style="list-style-type: none"> <li>Eigenvalues and eigenvectors</li> </ul>		§3.3 : 3, 8, 14, 16, 19, 21, 23, 27
6	Midterm week No lectures	<b>Midterm on Thursdy Oct 21 7-9pm</b>		
7	Oct. 25-29 (Lectures 17 to 19)	<b>Section 3.3</b> Diagonalization and eigenvalues. <ul style="list-style-type: none"> <li>Diagonalizing a matrix, diagonalizable matrix, diagonalization algorithm (<b>Omit A-invariants</b>)</li> </ul>	5	§3.3 : 3, 8, 14, 16, 19, 21, 23, 27
		<b>Section 4.4</b> Linear operators in $\mathbb{R}^3$ . <ul style="list-style-type: none"> <li>Reflections, projections and rotations</li> </ul>		§4.4 : 1 a) b), 3 a) c) f) g), 4 a), 5
8	Nov 1-5 (Lectures 20 to 22)	<b>Section 5.1</b> Subspaces and spanning. <ul style="list-style-type: none"> <li>Subspaces, null space, image space, eigenspace</li> <li>Spanning family (sets)</li> </ul>	6	§5.1 : 1, 2, 9, 16, 22.
		<b>Section 5.2</b> Independence and dimension <ul style="list-style-type: none"> <li>Linear impendence of a set</li> <li>Determining the dimension of a subspace</li> <li>Invertibility of A and linear independence</li> <li>Basis</li> </ul> (Start section 5.3. )		§5.2 : 2 a) b), 3, 4, 5 a), 6, 13, 17 a)

9	Nov 8-12 (Lectures 23 to 25)	<b>Section 5.3 Orthogonality</b> <ul style="list-style-type: none"> <li>• Dot-product, length of vectors, unit vector</li> <li>• Cauchy inequality</li> <li>• Orthogonal and orthonormal sets of vectors</li> <li>• Orthogonal basis</li> </ul>	7	§5.3 : 1, 2, 4, 5 a), 6, 10.
10	Nov. 15-19 (Lectures 26 to 28)	<b>Section 5.4 Rank of a matrix</b> <ul style="list-style-type: none"> <li>• Column space, Row space and their bases</li> <li>• Rank of a matrix, the Rank theorem</li> </ul>	8	§5.4 : 1 a), 2, 3, 7, 10
		<b>Section 5.5 Similarity and diagonalization</b> <ul style="list-style-type: none"> <li>• Similar matrices, symmetric matrices, trace of a matrix</li> <li>• Multiplicity of an eigenvalue,</li> <li>• Diagonalizability theorem</li> </ul>		§5.5 : 1, 5, 8, 9, 10
11	Nov. 22-26 (Lectures 29 to 31)	<b>Section 8.1.Orthogonal complements and projections</b> <ul style="list-style-type: none"> <li>• The Orthogonal lemma</li> <li>• Gram-Schmidt orthogonalization</li> <li>• Orthogonal complement of a subspace</li> </ul>	9	§8.1 : 1, 2, 3, 5, 9
		<b>Section 8.2 Orthogonal diagonalization</b> <ul style="list-style-type: none"> <li>• Orthogonal matrices,</li> <li>• Diagonalization of symmetric matrices..</li> </ul>		§8.2 : 1, 4, 5, 11
		<b>Section 6.1 Abstract vector spaces.</b> <ul style="list-style-type: none"> <li>• Axioms, examples and properties.</li> </ul>		§6.1 : 2, 4, 5
12	Nov. 29-Dec3 (Lectures 32 to 34)	<b>Section 6.2 Subspaces and spanning families</b> <ul style="list-style-type: none"> <li>• Recognizing subsets which are subspaces</li> <li>• Linear combinations</li> </ul>	10	§6.2 : 1, 2, 3, 4, 5, 7
		<b>Section 6.3 Linear independence and dimension</b> <ul style="list-style-type: none"> <li>• Linear independence and basis</li> </ul>		§6.3 : 1, 2, 4, 7
		<b>Section 6.4 Finite dimensional vector spaces</b> <ul style="list-style-type: none"> <li>• Completing LI sets to bases and reducing spanning families to bases.</li> <li>• Dimension</li> </ul>		§6.4 : 1, 2, 3, 18
13	Dec. 6 (Lecture 35)	<b>Appendix A : Complex numbers</b> <ul style="list-style-type: none"> <li>• Standard form, polar form, norm (absolute value)</li> <li>• Sums, products, quotient, DeMoivre's theorem</li> </ul>	11*	<b>Appendix A:</b> 1, 2, 3, 4, 5, 6, 18, 19, 20, 21 (Lab 11 posted with solutions only)

\*Lab 11\* will not be marked