Math 115 - Lab 4 - Fall 2010.

Lab for tutorial on Thanksgiving Monday. Topics:

- Properties of Invertible Matrices,
- Elementary Matrices,
- Matrix Transformations,
- Linear Transformations,
- Determinant

You are to provide full solutions to the following problems. You are allowed, and encouraged to collaborate with your classmates, use your notes and textbook and ask the TA for guidance. Direct copying of solutions is not encouraged, nor is it allowed or ethical.

Last name: ______________________ First name: ______________________

Student number: ______________________

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1. Let A be a $n \times n$ matrix and let I be the $n \times n$ identity matrix.
(a) If $A^2 = \mathbf{0}$, verify that $(I - A)^{-1} = I + A$.
(b) If $A^3 = \mathbf{0}$, verify that $(I - A)^{-1} = I + A + A^2$.

2. Let A and B denote invertible $n \times n$ matrices . Show that

$$A^{-1} + B^{-1} = A^{-1}(A + B)B^{-1}$$

.

3. In each case find an elementary matrix E such that $B = EA$.
(a) $A = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$.

(b) $A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}$.

4. Give the matrix of the transformation T in each case:
(a) $T : R^2 \rightarrow R^2$ is rotation through $\pi/4$.

(b) $T : R^3 \rightarrow R^3$ is reflection in the $X - Y$ plane.

5. Show that $T : R^2 \rightarrow R^2$ is not a linear transformation if

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ 0 \end{bmatrix}$$

6. Find an invertible matrix U such that $UA = R$ is in reduced row-echelon form, and express U as a product of elementary matrices, where

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 3 & 1 & 1 & 2 \\ 1 & -3 & 3 & 2 \end{bmatrix}$$

7. Compute the determinants of the following matrices.

(a) $det \begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix}$

(b) $det \begin{bmatrix} 3 & 1 & -5 & 2 \\ 1 & 3 & 0 & 1 \\ 1 & 0 & 5 & 2 \\ 1 & 1 & 2 & -1 \end{bmatrix}$