UNIVERSITY OF WATERLOO

MIDTERM EXAMINATION

FALL TERM 2008

Surname: _____

First Name: _____

Id.#: _____

C&O 370/CM 443	
Course Title	Deterministic OR Models
Instructor Henry Wolkowicz	9:30 MWF LEC 001
Date of Exam	October 17, 2008
Time Period	7-9pm, RCH 112 (make-up 5-7pm, RCH 208)
Number of Exam Pages (including this cover sheet)	11
Exam Type	Closed Book
Additional Materials Allowed	None (NO calculators)
Additional Instructions	Write your answers in the space provided.

Problem	Value	Mark Awarded
1	8	
2	10	
3	12	
4	15 (+5+2)	
TOTAL	45	

1 Dual Simplex Method

Consider the following minimization LP.

$$\begin{array}{rlrcl} z^* := & \min & 8x_1 + 5x_2 & & \\ & \text{s.t.} & x_1 + x_2 & \geq & 3 \\ & & 2x_1 + x_2 & \geq & 4 \\ & & x_1, x_2 & \geq & 0. \end{array}$$

Solve this problem using the dual simplex method. (Show your work clearly!)

1.1 Problem 1 cont...

2 Investment/Portfolio-Planning

An investor has decided to invest a total of $50,000 among three investment opportunities: savings certificates, municipal bonds, and stocks. The annual return on each investment is estimated to be 7%, 9%, and 14%, respectively. The investor does not intend to invest his annual interest returns (that is, he plans to use the interest to finance his hobbies). He would like to maximize his yearly return while investing a minimum of $10,000 in bonds. Also, the investment in stocks should not exceed the combined total investment in bonds and savings certificates. And, finally, he should invest between $5,000 and $15,000 in savings certificates.

1. Model the problem as an AMPL program. (Write appropriate lines for file.mod and file.dat. Carefully document the program.) Ensure that your problem is general enough so that it can extend easily if the number of investment opportunities changes, and also that it allows upper and lower bounds on each of the investment opportunities.

2. Extend the AMPL program so that an investment banker could use it to help several investors at once, where each investor has a separate total to invest. The banker should try and maximize the total sum of the returns (or a weighted sum of returns).

3. Suppose that there are several more investment opportunities. Add a constraint that would help *diversify* the portfolio, i.e. ensure that each investor invests in many investment opportunities rather than in just the ones with the high returns.

2.1 Problem 2 cont...

3 Parametric Programming

Consider the following parametric LP.

$$\begin{array}{rlrcl} z(\theta) := & \max & (1+\theta)x_1 + (4-\theta)x_2 & & \\ & \text{s.t.} & 2x_1 + x_2 & \leq & 10 \\ & & x_1 + x_2 & \leq & 6 \\ & & x_2 & \leq & 4 \\ & & x_1, x_2 & \geq & 0, \end{array}$$

where θ is a parameter. After adding 3 slack variables, and solving the problem with $\theta = 0$, we get the optimal basis $B = \{3, 1, 2\}$ and optimal inverse basis matrix $A_B^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$.

1. Compute the function $z(\theta)$ for all values of $\theta \geq 0$.
2. Compute a primal optimal solution $x^*(\theta)$, for all values of $\theta \geq 0$. Compute a dual optimal solution $y^*(\theta)$, for all values of $\theta \geq 0$.

3.1 Problem 3 cont...

3.2 Problem 3 cont...

4 Modelling - Cargo Plane

A cargo plane has three compartments for storing cargo: front, center, and back. These compartments have capacity limits on both *weight* and *space*, as summarized in table 1. Furthermore, the weight of

	Weight capacity (tons)	Space capacity (cu ft)
Front	12	7,000
Center	18	9,000
Back	10	5,000

Table 1: Capacity Limits

the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the airplane.

The following four cargoes in table 2 have been offered for shipment on an upcoming flight as space is available. Any portion of these cargoes can be accepted. The objective is to determine how much (if

Cargo	Total Weight (tons)	Volume (cu ft/ton)	Profit ($/ton)
1	20	500	280
2	16	700	360
3	25	600	320
4	13	400	250

Table 2: Cargos

any) of each cargo should be accepted and how to distribute each amont the compartments to maximize the profit for the flight.

1. Model the problem as an LP using mathematical notation.
2. **BONUS (5 MARKS)** Model the problem as an LP using AMPL notation with a file.mod and a file.dat.
3. **BONUS (2 MARKS)** Suppose that the shadow prices for the compartment capacity constraints ($3 \times 2 = 6$ constraints) are all positive. What can you say about the efficiency of the optimal solution?

4.1 Problem 4 cont...

4.2 Problem 4 cont...