summarized as follows:

$$x^*_{1,2} = 1 \quad \text{(basic)}$$
$$x^*_{1,3} = 7 \quad \text{(basic)}$$
$$x^*_{1,4} = 2 \quad \text{(nonbasic at lower bound)}$$
$$x^*_{3,2} = 2 \quad \text{(nonbasic at upper bound)}$$
$$x^*_{3,4} = 7 \quad \text{(nonbasic at lower bound)}$$
$$x^*_{4,2} = 4 \quad \text{(basic)}$$
$$x^*_4 = 0 \quad \text{(basic)}$$
$$z^* = 8(1) + 2(7) + 7(2) + 5(2) + 1(7) + 2(4) + 0(0) = 61$$

APPLICATIONS

Network flow problems have numerous applications. Also, many applications, although not exhibiting a pure network structure, have embedded network subproblems. This embedded network structure can be used to develop more efficient solution procedures for these problems by solving the subproblems utilizing the more efficient network simplex approach. This section presents a few of the more common applications of minimum-cost network flow problems.

Example 9.7: A Transshipment Problem

Consider a transportation system involving warehouses and retail outlets. Each warehouse i has s_i units of a particular commodity available, and each retail outlet j requires d_j units of the commodity. The objective is to find the shipping pattern that satisfies demands at minimum cost. Shipments can be made directly from a warehouse to a retail outlet or shipments can be made through intermediate warehouses. It is even possible to exchange shipments between retail outlets if this results in reduced shipping costs.

An example network containing three warehouses and four retail outlets is depicted in Figure 9.31. In addition to the supply and demand quantities, there is a unit shipping cost $c_{i,j}$ associated with each arc in the network. In the event that supply does not equal demand, the network can be balanced in the usual manner by adding a dummy warehouse node or a dummy retail outlet node. A simple variation of this problem, commonly referred to as the transportation problem, is studied in detail in Chapter 10.

Example 9.8: Production Scheduling

Consider again the production scheduling problem of Example 2.4, which is repeated here for convenience.

IMC, Inc., needs to schedule the monthly production of a certain item for the next 4 months. The unit production cost is estimated to be \$12 for the first 2 months