objective value for the perturbed problem reduce to the original unp... problem when $t = 0$. A systematic procedure for performing an analysis ... impact of variations in the c_j's can be summarized as follows.

Parametric programming procedure: c vector (maximization pr...

STEP 1. Set the parameter $t = 0$ and find an optimal solution to the original probl...

STEP 2. Add an additional top row to the optimal tableau containing the $z_j' - c_j'$ ... computed using $z_j' - c_j' = \mathbf{c}_B'\mathbf{B}^{-1}\mathbf{a}_j - c_j'$. The contribution to the objective ... value is given by $\mathbf{c}_B'\mathbf{B}^{-1}\mathbf{b}$.

STEP 3. Determine the parameter range over which the tableau is optimal by exami... optimality conditions

$$(z_j - c_j) + t(z_j' - c_j') \geq 0, \qquad \text{for all } j$$

Let this range be given by $l \leq t \leq u$, where l is the lower bound and u is the ... bound on parameter t. (Note that the values of l and u need not be finite...

STEP 4. If l is finite, determine which nonbasic variable has $(z_j - c_j) + t(z_j' - c_j') = ...$ $t = l$. Enter this variable into the basis by performing a primal simplex piv... will possibly result in a new tableau that is optimal for additional values of ...

Similarly, if u is finite, determine which nonbasic variable has $(z_j ...$ $t(z_j' - c_j') = 0$ when $t = u$. Enter this variable into the basis by performing ... simplex pivot. This will possibly result in a new tableau that is optimal for ... values of t.

STEP 5. Repeat Steps 3 and 4 until all the appropriate ranges of the parameter ha... investigated.

We now illustrate the solution procedure via the following simple ex...

Example 6.19: Parametric Programming: c Vector

$$\text{maximize } z = x_1 + 4x_2 + t(x_1 + x_2) = (1 + t)x_1 + (4 + t)x_2$$

subject to

$$2x_1 + x_2 \leq 10$$

$$x_1 + x_2 \leq 6$$

$$x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

Considering the original problem with $t = 0$ and denoting the respec... variables by x_3, x_4, and x_5 yield the optimal tableau shown in Table 6.30...

Note, from (6.295), that $\mathbf{c}$ and $\mathbf{c}'$ are given by

$$\mathbf{c} = (1 \quad 4 \quad 0 \quad 0 \quad 0)$$

$$\mathbf{c}' = (1 \quad -1 \quad 0 \quad 0 \quad 0)$$

Now, add an additional top row to the optimal tableau containing $z_j' - c_j' = ...$ c_j' with objective value $\mathbf{c}_B'\mathbf{B}^{-1}\mathbf{b}$. The updated tableau is shown in Table 6...

TABLE 6.30

	z	x_1	x_2	x_3	x_4	x_5	RHS
z	1	0	0	0	1	3	18
x_3	0	0	0	1	−2	1	2
x_1	0	1	0	0	1	−1	2
x_2	0	0	1	0	0	1	4

TABLE 6.31

	z	x_1	x_2	x_3	x_4	x_5	RHS
		0	0	0	1	−2	−2
z	1	0	0	0	1	3	18
x_3	0	0	0	1	−2	1	2
x_1	0	1	0	0	1	−1	2
x_2	0	0	1	0	0	1	4

Examining Table 6.31, we see that the present solution is $x = (2, 4, 2, 0, 0)$ and $z = 18 - 2t$. Now we use the optimality conditions given in (6.293) to determine for what range of the parameter t the current solution is optimal. The optimality conditions are

$$(z_4 - c_4) + t(z_4' - c_4') = 1 + t \geq 0$$

$$(z_5 - c_5) + t(z_5' - c_5') = 3 - 2t \geq 0$$

which result in

$$-1 \leq t \leq \tfrac{3}{2}$$

Thus, the current tableau is optimal for $-1 \leq t \leq \frac{3}{2}$. Note that $(z_4 - c_4) + t(z_4' - c_4') = 0$ when $t = -1$. Therefore, an alternative optimal solution exists for this tableau, which may be found by entering x_4 via a primal simplex pivot. The departing variable is x_1 and the new tableau is shown in Table 6.32.

TABLE 6.32

	z	x_1	x_2	x_3	x_4	x_5	RHS
		−1	0	0	0	−1	−4
z	1	−1	0	0	0	4	16
x_3	0	2	0	1	0	−1	6
x_4	0	1	0	0	1	−1	2
x_2	0	0	1	0	0	1	4

From Table 6.32, we see that it is optimal if

$$-1 - t \geq 0$$

$$4 - t \geq 0$$

That is, for $-\infty \leq t \leq -1$, $\mathbf{x} = (0, 4, 6, 2, 0)$ and $z = 16 - 4t$. Note that because we have already examined the bound $t = -1$, no finite bounds remain to be examined for the tableau of Table 6.32.

Returning to Table 6.31, we now examine the case of $t = \frac{3}{2}$. In this case, we perform a primal pivot entering x_5 into the basis. Table 6.33 shows the resulting tableau.

TABLE 6.33

	z	x_1	x_2	x_3	x_4	x_5	RHS
		0	0	2	−3	0	2
z	1	0	0	−3	7	0	12
x_5	0	0	0	1	−2	1	2
x_1	0	1	0	1	−1	0	4
x_2	0	0	1	−1	2	0	2

Examining Table 6.33, we find that $\mathbf{x} = (4, 2, 0, 0, 2)$ and $z = 12 + 2t$ for $\frac{3}{2} \leq t \leq \frac{7}{3}$. This results from the optimality conditions

$$-3 + 2t \geq 0$$

$$7 - 3t \geq 0$$

There are two finite bounds on the parameter t at this point, but we have already examined $t = \frac{3}{2}$. Thus, we look at $t = \frac{7}{3}$. For this case, we use a primal pivot to enter x_4 in Table 6.33. The resulting tableau is shown in Table 6.34.

TABLE 6.34

	z	x_1	x_2	x_3	x_4	x_5	RHS
		0	$\frac{3}{2}$	$\frac{1}{2}$	0	0	5
z	1	0	$-\frac{7}{2}$	$\frac{1}{2}$	0	0	5
x_5	0	0	1	0	0	1	4
x_1	0	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	5
x_4	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	1

Table 6

This results
At th
summarize
6.35. The r
graphically

$z = 16$

$$\tfrac{1}{2} + (\tfrac{1}{2})t \geq 0$$

This results in the solution $\mathbf{x} = (5, 0, 0, 1, 4)$, $z = 5 + 5t$ for $\frac{7}{3} \leq t \leq \infty$.

At this point, no *finite* bounds on t remain to be examined. Thus, we may summarize the results, for the entire range of t (from $-\infty$ to ∞), as shown in Table 6.35. The results of Table 6.35 pertaining to the objective value z can also be viewed graphically, as shown in Figure 6.10.

TABLE 6.35 RESULTS OF EXAMPLE 6.19

Range of t	Optimal solution	Optimal objective
$-\infty \leq t \leq -1$	$\mathbf{x} = (0, 4, 6, 2, 0)$	$z = 16 - 4t$
$-1 \leq t \leq \frac{3}{2}$	$\mathbf{x} = (2, 4, 2, 0, 0)$	$z = 18 - 2t$
$\frac{3}{2} \leq t \leq \frac{7}{3}$	$\mathbf{x} = (4, 2, 0, 0, 2)$	$z = 12 + 2t$
$\frac{7}{3} \leq t \leq \infty$	$\mathbf{x} = (5, 0, 0, 1, 4)$	$z = 5 + 5t$

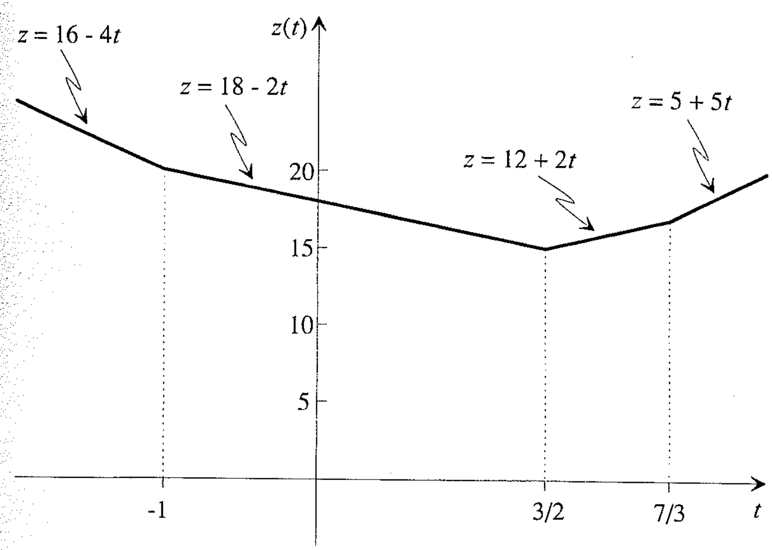

Figure 6.10 Graph for Example 6.19.