C&O 350 Linear Optimization – Fall 2005

Assignment 3 **Due: October 6th, 2005 (Thu), 3:00PM**

From the notes do the following exercises

- 4.8.7 (page 56),

Exercise 1: Suppose we are given m different numbers $\{a_1, \ldots, a_m\}$. Consider the following linear program (P), which has one variable only,

$$\begin{array}{ll} \min & x \\ \text{subject to} & \\ & x \geq a_i \quad (i = 1, \ldots, m) \end{array}$$

1. Find the dual (D) of (P),
2. Write the complementary slackness conditions for (P) and (D),
3. Suppose y^* is an optimal solution to (D), prove using complementary slackness theory that, for $i = 1, \ldots, m$,

$$y_i^* = \begin{cases} 1 & \text{if } a_i \text{ is the } \textit{largest} \text{ number among } \{a_1, \ldots, a_m\} \\ 0 & \text{otherwise} \end{cases}$$

Exercise 2: Let (P) be a linear program and let (D) be its dual. Let x^* be a feasible solution for (P) and let y^* be a feasible solution for (D). Show that if all variables of (P) and (D) are free (unrestricted) then x^* is an optimal solution to (P) and y^* is an optimal solution to (D).

Exercise 3: Prove Theorem 4.9 for the case where (P) is in standard *inequality* form. IMPORTANT: your proof should be self contained. In particular you should NOT convert (P) to a problem in standard equality form and apply Theorem 4.7.