# Recap

Definition of LP problem: (Pg 9)

minimizing or maximizing a linear function subject to finite number of linear equalities and/or linear inequalities.

Examples

$$\begin{array}{rrrrrrr} \text{maximize} & 2x_1 & + & 3x_2 & & \\ \text{subject to} & & & & & \\ & 2x_1 & + & x_2 & \leq & 10 \\ & x_1 & + & x_2 & \leq & 6 \\ & -x_1 & + & x_2 & \leq & 4 \\ & x_1 & , & x_2 & \geq & 0 \end{array}$$

$$\begin{array}{lrcll} \text{maximize} & \sum_{j=1}^{n} c_j x_j & & & \\ \text{subject to} & \sum_{j=1}^{n} a_{ij} x_j & \leq & b_i & (i = 1, 2, \ldots, m) \\ & x_j & \geq & 0 & (j = 1, 2, \ldots, n) \end{array}$$

More examples of LP: See page 9 of notes.

## Terminology (Pg 10)

Example: The Orange Factory Problem

$$\begin{array}{llrcrcr} \text{maximize} & & 2x_1 & + & 3x_2 & & \\ \text{subject to} & & 2x_1 & + & x_2 & \leq & 10 \\ & & x_1 & + & x_2 & \leq & 6 \\ & & -x_1 & + & x_2 & \leq & 4 \\ & & x_1 & , & x_2 & \geq & 0 \end{array}$$

⟸ Objective function (maximize $2x_1 + 3x_2$)

⟸ Constraints (subject to ...)

(Defn) **Feasible solution**

Assignment of values to $x_j$'s that satisfies ALL constraints.

E.g., $[4,1]^T$, $[2.1,3]^T$, $[1,5]^T$.

(Defn) **Feasible region**: Set of feasible solutions.

(Defn) **Objective value**

Value of objective function at given solution.

E.g., obj. value of $[4,1]^T$ is 11.

(Defn) **Optimal solution**

Feasible solution with best objective value.

(Defn) **Optimal value**

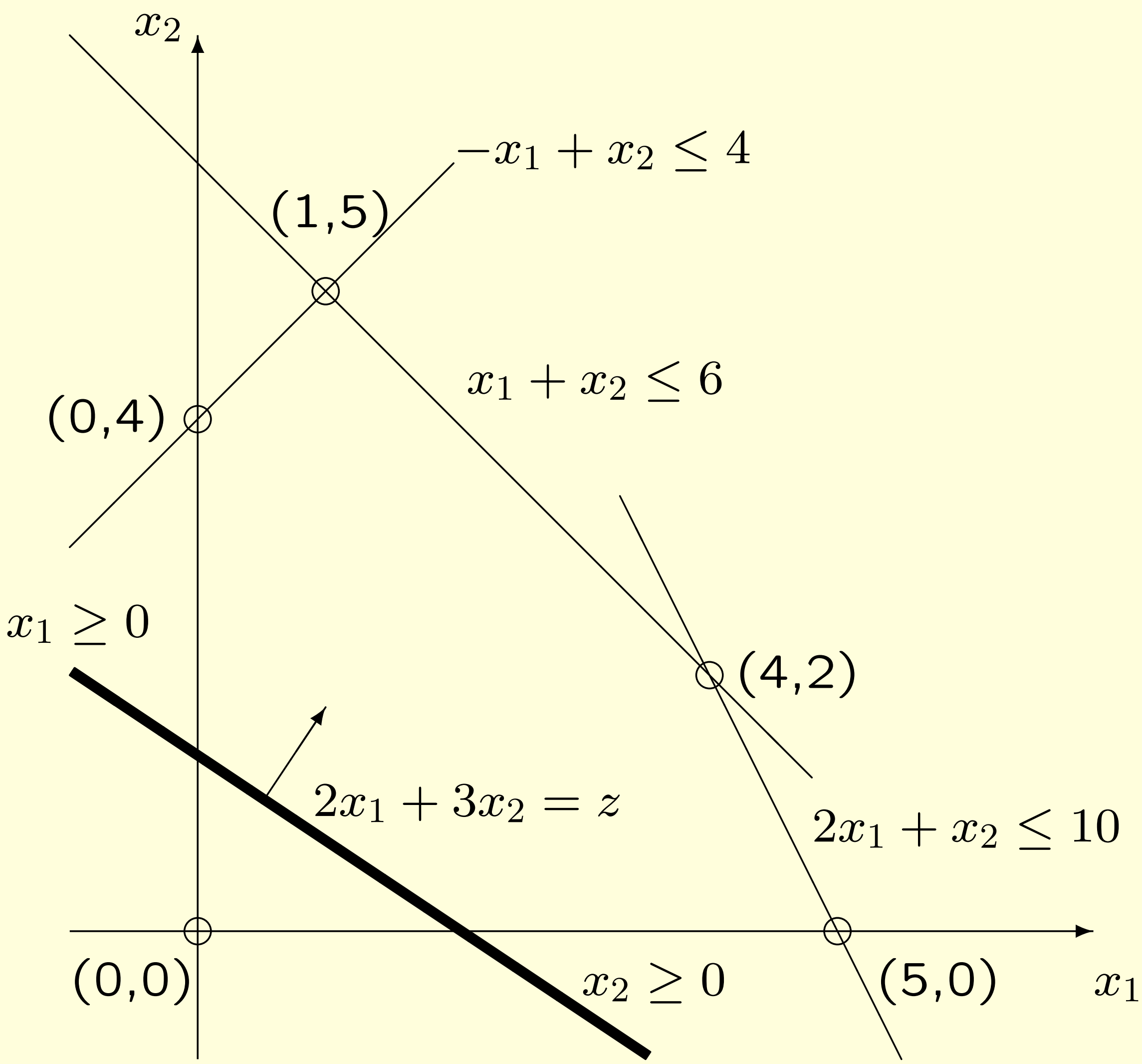
Objective value of an optimal solution (if exists).

E.g., $[1,5]^T$ is an opt. solution.

Opt. value of the LP is 17.

# Geometry

Example (The Orange Factory Problem, Pg 11)




Geometrically the problem is

> find the largest value of $z$ so that the line $2x_1 + 3x_2 = z$ intersects the feasible region, and find a point of intersection.

# Some More Examples

A transportation problem

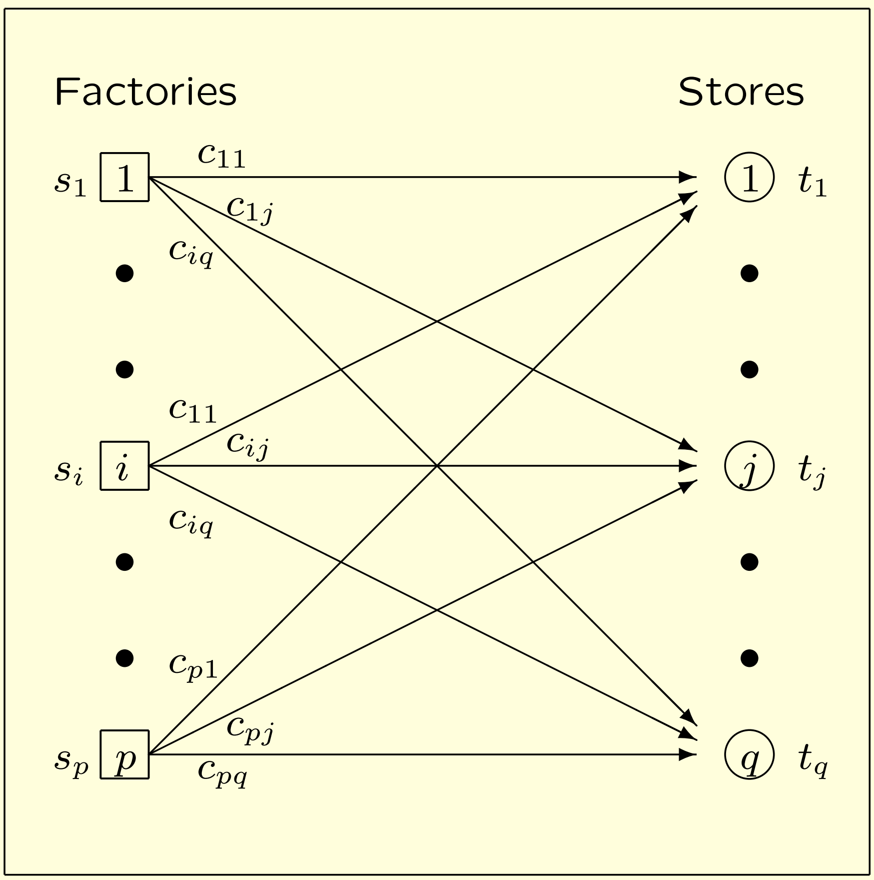
**One** product is made in $p$ factories and sold in $q$ stores.

Factory $i$ produces $s_i$ units/month.

Store $j$ orders $t_j$ units/month.

Cost $c_{ij}$ dollars to ship a unit from factory $i$ to store $j$.

Problem: Find least expensive way to transport product.




Picture NOT in notes

Decision variables:

$x_{ij}$ = amount to ship from factory $i$ to store $j$

Objective: minimize cost $= \sum_{i=1}^{p} \sum_{j=1}^{q} c_{ij} x_{ij}$

Constraints:

$$\begin{array}{lrcll}
\text{(Factory } i\text{)} & \sum_{j=1}^{q} x_{ij} & = & s_i & (i = 1, 2, \ldots, p) \\
\text{(Store } j\text{)} & \sum_{i=1}^{p} x_{ij} & = & t_i & (j = 1, 2, \ldots, q) \\
\text{(Route } (i,j)\text{)} & x_{ij} & \geq & 0 & \begin{pmatrix} i = 1, 2, \ldots, p, \\ j = 1, 2, \ldots, q \end{pmatrix}
\end{array}$$

<u>Mathematical model</u>

$$\begin{array}{llrcll}
\text{minimize} & \sum_{i=1}^{p} \sum_{j=1}^{q} c_{ij} x_{ij} & & & & \\
\text{subject to} & & \sum_{j=1}^{q} x_{ij} & = & s_i & (i = 1, 2, \ldots, p) \\
& & \sum_{i=1}^{p} x_{ij} & = & t_i & (j = 1, 2, \ldots, q) \\
& & x_{ij} & \geq & 0 & \begin{pmatrix} i = 1, 2, \ldots, p, \\ j = 1, 2, \ldots, q \end{pmatrix}
\end{array}$$

This LP problem does not have any feasible solution unless

$$\sum_{i=1}^{p} s_i = \sum_{j=1}^{q} t_j$$

Exercise: Show that the LP problem has a feasible solution when the above equation holds.