Q1

Find the minimum polynomial of A, and thus determine whether or not A is diagonalisable.

$$A = \begin{bmatrix} 3 & 4 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}.$$

Q2

Find the minimum polynomials of A and B (a and b are real numbers)

$$A = \begin{bmatrix} 2 & a \\ 0 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & a & 0 \\ 0 & 3 & b \\ 0 & 0 & 3 \end{bmatrix}$$

Q3

Let A be a real $n \times n$ matrix and suppose $A^2 = A$.

(i) Assuming that $A \neq I_n$ and $A \neq 0_n$, find the minimum polynomial of A.

(ii) Prove that A is not invertible.

(iii) Prove that A is diagonalisable.

(iv) What are the possible choices for the characteristic polynomial of A?

Q4

Show that A and B have different characteristic polynomials, but have the same minimum polynomial.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$