Sensor Network Localization, Euclidean Distance Matrix Completions, and Graph Realization

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Abstract

Wireless sensor networks have many applications, e.g. in monitoring physical or environmental conditions (temperature, sound, vibration, pressure, battlefield surveillance, home automation, hospital patients, traffic control, etc.). The sensor network localization, $\boldsymbol{SNL}$, problem consists of locating the positions of ad hoc wireless sensors, given only the distances between sensors that are within radio range and the positions of a subset of the sensors (called anchors). One main point is to view $\boldsymbol{SNL}$ as a (nearest) Euclidean Distance Matrix, $\boldsymbol{EDM}$, completion problem that does not distinguish between the anchors and the sensors. We show that there are advantages for using the well-studied $\boldsymbol{EDM}$ model. This problem can be relaxed to a weighted, nearest, (positive) semidefinite programming, $\boldsymbol{SDP}$, completion problem. This relaxation is ill-conditioned in two ways. First, it is, implicitly, highly degenerate in the sense that the feasible set is restricted to a low dimensional face of the $\boldsymbol{SDP}$ cone. This means that the Slater constraint qualification fails. Second, nonuniqueness of the optimal solution results in large sensitivity to small perturbations in the data.

The degeneracy in the $\boldsymbol{SDP}$ arises from cliques in the graph of the $\boldsymbol{SNL}$ problem. We take advantage of the absence of the Slater constraint qualification and derive a preprocessing technique that solves the $\boldsymbol{SNL}$ problem. With exact data, we explicitly solve the corresponding $\boldsymbol{SDP}$ problem without using any $\boldsymbol{SDP}$ solver. We do this by finding explicit representations of the faces of the $\boldsymbol{SDP}$ cone corresponding to intersections of cliques of the $\boldsymbol{SNL}$ problem. For problems with noise, we first solve nearest matrix problems to get best $\boldsymbol{EDM}$ approximations.