Explicit Sensor Network Localization using Semidefinite Programming and Clique Reductions

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Outline

1. Preliminaries
   - The Sensor Network Localization, SNL, Problem
   - Underlying Graph Realization/Partial EDM
   - Semidefinite Programming (SDP); Facial Structure
   - Further Notation

2. Clique/Facial Reduction (exploit degeneracy)
   - Single Clique Reduction (Exploit Degeneracy)
   - Two Clique Reduction and EDM Completion
   - Reduction and Completion; Singular Case

3. Algorithm
   - Clique Unions and Node Absorptions
   - Results (low CPU time; high accuracy)

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Sensor Network Localization, SNL, Problem

Wireless Sensor Network

- n ad hoc wireless sensors (nodes) to locate in $\mathbb{R}^r$,
  (r is embedding dimension;
  sensors $p_i \in \mathbb{R}^r, i \in V := 1, \ldots, n$)
- m of the sensors are anchors, $p_i, i = n - m + 1, \ldots, n$)
  (e.g. using GPS)
- pairwise distances known within radio range R,
  $D_{ij} = \|p_i - p_j\|^2, ij \in E$

Current Techniques: Nearest, Weighted, SDP Approx.

$\min_{Y \succeq 0, Y \in \Omega} \|H \circ (\mathcal{K}(Y) - D)\|$
SDP program is: Expensive/low accuracy/implicitly highly degenerate (cliques restrict ranks of feasible Ys)

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Applications

Tracking Humans/Animals/Equipment

- monitoring: natural habitat; earthquakes and volcanos; weather and ocean currents.
- military, tracking of goods, vehicle positions, surveillance, (where open-air positioning is not feasible), random deployment in inaccessible terrains or disaster relief operations

Future Applications?

- bicycles, car/computer parts, guns (to prevent theft)
- students/teenagers (prevent class absence)

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Underlying Graph Realization/Partial EDM

Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \omega)$

- node set $\mathcal{V} = \{1, \ldots, n\}$
- edge set $(i,j) \in \mathcal{E}$; $\omega_{ij} = \|p_i - p_j\|^2$ known approximately
- The anchors form a clique (complete subgraph)
- Realization of $\mathcal{G}$ in $\Re^r$: a mapping of node $v_i \to p_i \in \Re^r$ with squared distances given by ω.

Corresponding Partial Euclidean Distance Matrix, EDM

$$D_{ij} = \begin{cases} d_{ij}^2 & \text{if } (i,j) \in \mathcal{E} \\ 0 & \text{otherwise,} \end{cases}$$

$d_{ij}^2 = \omega_{ij}$ are known squared Euclidean distances between sensors p_i, p_j; anchors correspond to a clique.

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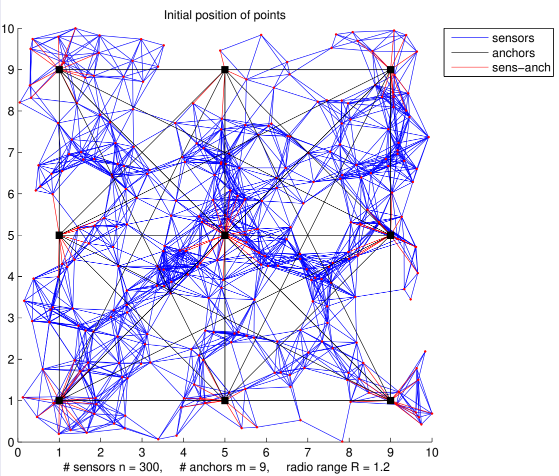
Sensor Localization Problem/Partial EDM

Sensors ○ and Anchors ■



SDP: Cone and Faces

$\mathcal{S}^k_+$, Cone of SDP matrices in $\mathcal{S}^k$

- inner product $\langle A, B \rangle = \operatorname{trace} AB$
- Löwner (psd) partial order $A \succeq B, A \succ B$

Faces of cone K

- $F \subseteq K$ is a face of K, denoted $\boxed{F \trianglelefteq K}$, if $\left(x, y \in K, \ \frac{1}{2}(x+y) \in F\right) \implies \left(\operatorname{cone}\{x, y\} \subseteq F\right)$.
- $\boxed{F \lhd K}$, if $F \trianglelefteq K$, $F \neq K$; F is $\boxed{\text{proper face}}$ if $\{0\} \neq F \lhd K$.
- $F \trianglelefteq K$ is $\boxed{\text{exposed}}$ if: intersection of K with a hyperplane.
- $\boxed{\operatorname{face}(S)}$ denotes smallest face of K that contains set S.

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Facial Structure of SDP Cone

$\mathcal{S}^k_+$ is a Facially Exposed Cone

All faces are exposed.

Faces of $\mathcal{S}^k_+$ Equivalence to Subspaces of $\mathbb{R}^k$

- face $F \trianglelefteq \mathcal{S}^k_+$ is determined by range of any, $S \in \operatorname{relint} F$,
  i.e. let $S = U\Gamma U^T$ be compact spectral decomposition, with diagonal matrix of eigenvalues $\Gamma \in \mathcal{S}^t_{++}$, then:
  $\boxed{F = U\mathcal{S}^t_+ U^T}$ (F associated with $\mathcal{R}(U)$)
- $\dim F = t(t+1)/2$.

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Further Notation

Matrix with Fixed Principal Submatrix

For $Y \in \mathcal{S}^n$, $\alpha \subseteq \{1, \ldots, n\}$: $Y[\alpha]$ denotes principal submatrix formed from rows & cols with indices α.

Sets with Fixed Principal Submatrices

If $|\alpha| = k$ and $\bar{Y} \in \mathcal{S}^k$, then:
$\mathcal{S}^n(\alpha, \bar{Y}) := \{ Y \in \mathcal{S}^k : Y[\alpha] = \bar{Y} \}$,
$\mathcal{S}^n_+(\alpha, \bar{Y}) := \{ Y \in \mathcal{S}^k_+ : Y[\alpha] = \bar{Y} \}$
i.e. the subset of matrices $Y \in \mathcal{S}^k$ ($Y \in \mathcal{S}^k_+$) with principal submatrix $Y[\alpha]$ fixed to $\bar{Y}$.

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SNL Connection to SDP (Lin. Trans., Adjoints)

$v = \mathrm{diag}\,(S) \in \mathbb{R}^n$, $S = \mathrm{Diag}\,(v) \in \mathcal{S}^n$

$\mathrm{diag} = \mathrm{Diag}^*$ (adjoint)
for $B \in \mathcal{S}^n$, $\mathrm{offDiag}\,(B) := B - \mathrm{Diag}\,(\mathrm{diag}\,(B))$

$D = \mathcal{K}\,(B) \in \mathcal{E}^n$, $B = \mathcal{K}^\dagger(D) \in \mathcal{S}^n \cap \mathcal{S}_C$

Let $P^T = \begin{bmatrix} p_1 & p_2 & \dots & p_n \end{bmatrix} \in \mathcal{M}^{r \times n}$;
$B := PP^T \in \mathcal{S}^n$; $D \in \mathcal{E}^n$ be corresponding EDM.

$$
\begin{array}{rcl}
(\text{from } \mathcal{S}^n) \quad \mathcal{K}\,(B) & := & \mathcal{D}_e(B) - 2B \\
& := & \mathrm{diag}(B)\,e^T + e\,\mathrm{diag}(B)^T - 2B \\
& = & \left(p_i^T p_i + p_j^T p_j - 2p_i^T p_j\right)_{i,j=1}^n \\
& = & \left(\|p_i - p_j\|_2^2\right)_{i,j=1}^n \\
& = & D \quad (\text{to } \mathcal{E}^n).
\end{array}
$$

$\mathcal{K} : \mathcal{S}^n_+ \cap \mathcal{S}_C \to \mathcal{E}^n$

Linear Transformations: $\mathcal{D}_v(B), \mathcal{K}(B), \mathcal{T}(D)$

- allow: $\mathcal{D}_v(B) := \operatorname{diag}(B)\, v^T + v \operatorname{diag}(B)^T$;
  $\mathcal{D}_v(y) := yv^T + vy^T$
- adjoint $\mathcal{K}^*(D) = 2(\operatorname{Diag}(De) - D)$.
- $\mathcal{K}$ is $1-1$, onto between centered and hollow subspaces:
  $\mathcal{S}_C := \{B \in \mathcal{S}^n : Be = 0\}$;
  $\mathcal{S}_H := \{D \in \mathcal{S}^n : \operatorname{diag}(D) = 0\} = \mathcal{R}(\operatorname{offDiag})$
- $J := I - \frac{1}{n} ee^T$ (orthogonal projection onto $M := \{e\}^\perp$);
- $\boxed{\mathcal{T}(D) := -\frac{1}{2} J \operatorname{offDiag}(D) J \qquad (= \mathcal{K}^\dagger(D))}$

Properties of Linear Transformations

$\mathcal{K}, \mathcal{T}, \mathrm{Diag}, \mathcal{D}_e$

$$\mathcal{R}(\mathcal{K}) = \mathcal{S}_H; \qquad \underline{\mathcal{N}(\mathcal{K}) = \mathcal{R}(\mathcal{D}_e)};$$

$$\mathcal{R}(\mathcal{K}^*) = \mathcal{R}(\mathcal{T}) = \mathcal{S}_C; \qquad \mathcal{N}(\mathcal{K}^*) = \mathcal{N}(\mathcal{T}) = \mathcal{R}(\mathrm{Diag});$$

$$\mathcal{S}^n = \mathcal{S}_H \oplus \mathcal{R}(\mathrm{Diag}) = \mathcal{S}_C \oplus \mathcal{R}(\mathcal{D}_e).$$

$$\mathcal{T}(\mathcal{E}^n) = \mathcal{S}^n_+ \cap \mathcal{S}_C \quad \text{and} \quad \mathcal{K}(\mathcal{S}^n_+ \cap \mathcal{S}_C) = \mathcal{E}^n.$$

Single Clique/Facial Reduction

$\bar{D} \in \mathcal{E}^k$, $\alpha \subseteq 1:n$, $|\alpha| = k$

Define $\mathcal{E}^n(\alpha, \bar{D}) := \left\{D \in \mathcal{E}^n : D[\alpha] = \bar{D}\right\}$.

Given $\bar{D}$; find a corresponding $B \succeq 0$; find the corresponding face; find the corresponding subspace.

Basic Theorem for Single Clique/Facial Reduction

THEOREM 1: Single Clique Reduction

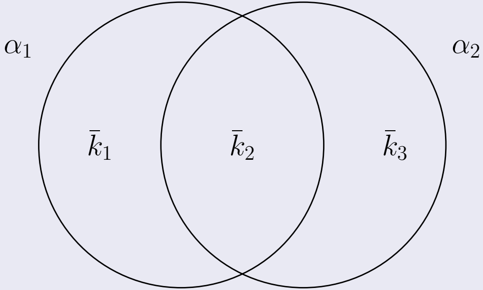
Let $\bar{D} := D[1:k] \in \mathcal{E}^k$, $k < n$, with embedding dimension $t \leq r$, and $B := \mathcal{K}^\dagger(\bar{D}) = \bar{U}_B S \bar{U}_B^T$, where $\bar{U}_B \in \mathcal{M}^{k\times t}$, $\bar{U}_B^T \bar{U}_B = I_t$, and $S \in \mathcal{S}^t_{++}$. Furthermore, let $U_B := \begin{bmatrix} \bar{U}_B & \frac{1}{\sqrt{k}} e \end{bmatrix} \in \mathcal{M}^{k\times(t+1)}$, $U := \begin{bmatrix} U_B & 0 \\ 0 & I_{n-k} \end{bmatrix}$, and let $\begin{bmatrix} V & \frac{U^T e}{\|U^T e\|} \end{bmatrix} \in \mathcal{M}^{n-k+t+1}$ be orthogonal. Then:

$$\boxed{\begin{aligned} \operatorname{face} \mathcal{K}^\dagger\left(\mathcal{E}^n(1:k, \bar{D})\right) &= \left(U \mathcal{S}_+^{n-k+t+1} U^T\right) \cap \mathcal{S}_C \\ &= (UV) \mathcal{S}_+^{n-k+t} (UV)^T \end{aligned}}$$

Note that we add $\frac{1}{\sqrt{k}} e$ to represent $\mathcal{N}(\mathcal{K})$; then we use V to eliminate e to recover *centered* face.

Positive Integers for Intersecting Cliques

Two Intersection Sets, α_1, α_2.



For each clique $|\alpha| = k$, we get a corresponding face/subspace ($k \times r$ matrix) representation. We now see how to handle two cliques, α_1, α_2, that intersect.

Two (Intersecting) Clique Reduction/Subsp. Repres.

THEOREM 2: Clique Intersection Reduction/Subsp. Repres.

$$\alpha_1 := 1\!:\!(\bar{k}_1+\bar{k}_2), \quad \alpha_2 := (\bar{k}_1+1)\!:\!(\bar{k}_1+\bar{k}_2+\bar{k}_3) \quad \subseteq \quad 1\!:\!n,$$
$$k_1 := |\alpha_1| = \bar{k}_1+\bar{k}_2, \quad k_2 := |\alpha_2| = \bar{k}_2+\bar{k}_3,$$
$$k := \bar{k}_1+\bar{k}_2+\bar{k}_3.$$

For $i=1,2$, let $\bar{D}_i := D[\alpha_i] \in \mathcal{E}^{k_i}$, with embedding dimension t_i, and $B_i := \mathcal{K}^\dagger(\bar{D}_i) = \bar{U}_i S_i \bar{U}_i^T$, where $\bar{U}_i \in \mathcal{M}^{k_i \times t_i}$, $\bar{U}_i^T \bar{U}_i = I_{t_i}$, and $S_i \in \mathcal{S}^{t_i}_{++}$. Furthermore, for $i=1,2$, let $U_i := \begin{bmatrix} \bar{U}_i & \frac{1}{\sqrt{k_i}} e \end{bmatrix} \in \mathcal{M}^{k_i \times (t_i+1)}$, and let $\bar{U} \in \mathcal{M}^{k \times (t+1)}$ satisfy

$$\boxed{\mathcal{R}(\bar{U}) = \mathcal{R}\left(\begin{bmatrix} U_1 & 0 \\ 0 & I_{\bar{k}_3} \end{bmatrix}\right) \cap \mathcal{R}\left(\begin{bmatrix} I_{\bar{k}_1} & 0 \\ 0 & U_2 \end{bmatrix}\right), \text{ with } \bar{U}^T\bar{U} = I_{t+1}}$$

cont...

Two (Intersecting) Clique Reduction, cont...

THEOREM 2 Nonsing. Clique Inters. cont...

cont... with

$$\boxed{\mathcal{R}(\bar{U}) = \mathcal{R}\left(\begin{bmatrix} U_1 & 0 \\ 0 & I_{\bar{k}_3}\end{bmatrix}\right) \cap \mathcal{R}\left(\begin{bmatrix} I_{\bar{k}_1} & 0 \\ 0 & U_2\end{bmatrix}\right), \text{ with } \bar{U}^T\bar{U} = I_{t+1}},$$

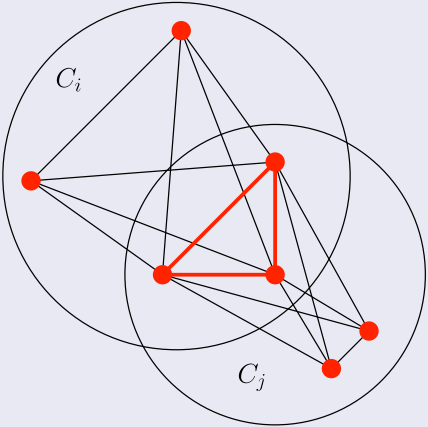
let $U := \begin{bmatrix} \bar{U} & 0 \\ 0 & I_{n-k}\end{bmatrix} \in \mathcal{M}^{n\times(n-k+t+1)}$ and let

$\begin{bmatrix} V & \frac{U^Te}{\|U^Te\|}\end{bmatrix} \in \mathcal{M}^{n-k+t+1}$ be orthogonal. Then

$$\boxed{\begin{aligned} \underline{\textstyle\bigcap_{i=1}^{2} \text{face}\, \mathcal{K}^{\dagger}\left(\mathcal{E}^n(\alpha_i, \bar{D}_i)\right)} &= \left(U\mathcal{S}_+^{n-k+t+1}U^T\right) \cap \mathcal{S}_C \\ &= (UV)\mathcal{S}_+^{n-k+t}(UV)^T \end{aligned}}$$

Basic work: find $\bar{U}$ to repres. inters. of 2 subspaces.

Two (Intersecting) Clique Reduction Figure



Completion: missing distances can be recovered if desired.

Two (Intersecting) Clique Explicit Completion

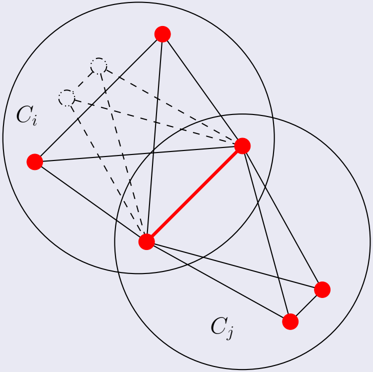
COR. Intersection with Embedding Dim. r/Completion

Hypotheses of Theorem 2 holds. Let $\bar{D}_i := D[\alpha_i] \in \mathcal{E}^{k_i}$, for $i = 1,2$, $\beta \subseteq \alpha_1 \cap \alpha_2, \gamma := \alpha_1 \cup \alpha_2, \bar{D} := D[\beta], B := \mathcal{K}^\dagger(\bar{D})$, $\quad \bar{U}_\beta := \bar{U}(\beta,:)$, where $\bar{U} \in \mathcal{M}^{k\times(t+1)}$ satisfies intersection equation of Theorem 2. Let $\begin{bmatrix} \bar{V} & \frac{\bar{U}^T e}{\|\bar{U}^T e\|} \end{bmatrix} \in \mathcal{M}^{t+1}$ be orthogonal. Let $\boxed{Z := (J\bar{U}_\beta \bar{V})^\dagger B((J\bar{U}_\beta \bar{V})^\dagger)^T}$. If the embedding dimension for $\bar{D}$ is r, THEN $t = r$ in Theorem 2, and $Z \in \mathcal{S}^r_+$ is the unique solution of the equation $(J\bar{U}_\beta \bar{V})Z(J\bar{U}_\beta \bar{V})^T = B$, and the exact completion is

$$\boxed{D[\gamma] = \mathcal{K}\left((\bar{U}\bar{V})Z(\bar{U}\bar{V})^T\right)}.$$

2 (Inters.) Clique Red. Figure/Singular Case

Two (Intersecting) Clique Reduction Figure/Singular Case



Use R as lower bound in singular/nonrigid case.

Two (Inters.) Clique Explicit Compl.; Sing. Case

COR. Clique-Sing.; Intersect. Embedding Dim. $r-1$

Hypotheses of previous COR holds. For $i = 1, 2$, let $\beta \subset \delta_i \subseteq \alpha_i$, $A_i := J\bar{U}_{\delta_i}\bar{V}$, where $\bar{U}_{\delta_i} := \bar{U}(\delta_i, :)$, and $B_i := \mathcal{K}^\dagger(D[\delta_i])$. Let $\bar{Z} \in \mathcal{S}^t$ be a particular solution of the linear systems

$$\begin{array}{rcl} A_1 Z A_1^T & = & B_1 \\ A_2 Z A_2^T & = & B_2. \end{array}.$$

If the embedding dimension of $D[\delta_i]$ is r, for $i = 1, 2$, but the embedding dimension of $\bar{D} := D[\beta]$ is $r-1$, then the following holds. cont...

2 (Inters.) Clique Expl. Compl.; Degen. cont. . .

COR. Clique-Degen. cont. . .

The following holds:

1. $\dim \mathcal{N}(A_i) = 1$, for $i = 1, 2$.
2. For $i = 1, 2$, let $n_i \in \mathcal{N}(A_i)$, $\|n_i\|_2 = 1$, and $\Delta Z := n_1 n_2^T + n_2 n_1^T$. Then, Z is a solution of the linear systems if and only if
$Z = \bar{Z} + \tau \Delta Z, \quad$ for some $\tau \in \mathcal{R}$
3. There are at most two nonzero solutions, τ_1 and τ_2, for the generalized eigenvalue problem $-\Delta Z v = \tau \bar{Z} v$, $v \neq 0$. Set $Z_i := \bar{Z} + \frac{1}{\tau_i} \Delta Z$, for $i = 1, 2$. Then the exact completion is one of $\boxed{D[\gamma] \in \left\{\mathcal{K}\left(\bar{U}\bar{V}Z_i\bar{V}^T\bar{U}^T\right) : i = 1, 2\right\}}$

Algorithm

Initialize

$C_i := \{j : (D_p)_{ij} < (R/2)^2\}$, for $i = 1, \ldots, n$

Iterate

- For $|C_i \cap C_j| \geq r+1$, do Rigid Clique Union
- For $|C_i \cap \mathcal{N}(j)| \geq r+1$, do Rigid Node Absorption
- For $|C_i \cap C_j| = r$, do Non-Rigid Clique Union (lower bnds)
- For $|C_i \cap \mathcal{N}(j)| = r$, do Non-Rigid Node Absorp. (lower bnds)

Finalize

When $\exists$ a clique containing all anchors, use computed facial representation and positions of anchors to solve for X

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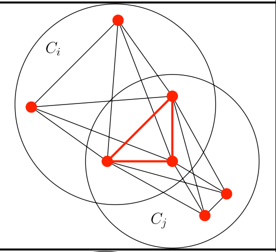
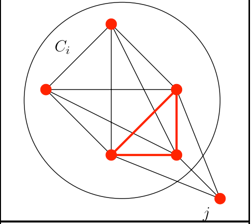
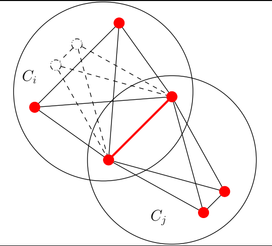
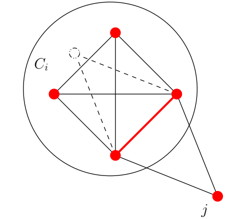
	Clique Union	Node Absorption
Rigid		
Non-rigid		

Results

Rigid Clique Union

n / R	0.7	0.6	0.5	0.4
2000	1	7	91	362
4000	1	1	1	16
6000	1	1	1	1
8000	1	1	1	1
10000	1	1	1	1

Remaining Cliques

n / R	0.7	0.6	0.5	0.4
2000	4.8	4.6	4.2	4.1
4000	9.2	9.4	9.1	9.2
6000	16.0	14.7	15.3	14.9
8000	22.9	22.5	20.9	21.0
10000	38.3	32.7	29.1	30.7

CPU Seconds

n / R	0.7	0.6	0.5	0.4
2000	-10.1	-10.8	$-$	$-$
4000	-10.9	-11.0	-10.5	-9.6
6000	-11.6	-10.7	-10.6	-10.0
8000	-11.1	-11.0	-10.7	-9.2
10000	-11.0	-11.0	-10.2	-10.4

Max log(Error)

Rigid Clique Union and Node Absorption

n / R	0.7	0.6	0.5	0.4
2000	1	1	2	78
4000	1	1	1	1
6000	1	1	1	1
8000	1	1	1	1
10000	1	1	1	1

Remaining Cliques

n / R	0.7	0.6	0.5	0.4
2000	4.9	4.9	6.1	13.2
4000	9.2	9.5	9.1	9.8
6000	16.1	15.1	15.1	14.8
8000	22.7	22.4	21.0	21.3
10000	32.5	32.4	28.8	30.6

CPU Seconds

n / R	0.7	0.6	0.5	0.4
2000	-10.1	-10.8	-9.8	-8.8
4000	-10.9	-11.0	-10.5	-9.6
6000	-11.6	-10.7	-10.6	-10.0
8000	-11.1	-11.0	-10.7	-9.2
10000	-11.0	-11.0	-10.2	-10.4

Max log(Error)

Summary

- SDP relaxation of SNL is highly (implicitly) degenerate: The feasible set of this SDP is restricted to a low dim. face of the SDP cone, causing the Slater constraint qualification (strict feasibility) to fail
- We take advantage of this degeneracy by finding explicit representations of intersections of faces of the SDP cone corresponding to unions of intersecting cliques
- Without using an SDP-solver (eg. SeDuMi or SDPT3), we quickly compute the exact solution to the SDP relaxation (except for round-off error from computing eigenvectors, etc.)

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Thanks for your attention!

Explicit Sensor Network Localization using Semidefinite Programming and Clique Reductions

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