Strong Duality, Complementarity, and Duality Gaps in Conic Convex Optimization

Simon Schurr[a], Levent Tunçel[a], and Henry Wolkowicz[a]

[a]University of Waterloo
Dept. of Combinatorics & Optimization
Waterloo, Ontario Canada

For convex minimization problems with optimal value v_P, *weak duality* holds in general, i.e. the optimal value of the Lagrangian dual provides a lower bound, $v_D \leq v_P$. However, *strong duality* (i.e. $v_D = v_P$ and v_D is attained) requires a *constraint qualification, CQ*, e.g. the Slater (strict feasibility) CQ. In the absence of a CQ, the dual optimal value v_D may be unattained and/or there can be a positive duality gap $v_P - v_D > 0$. In addition, the (near) absence of a CQ results in numerical difficulties.

We first present known and new conditions that guarantee strong duality without any constraint qualification. These are based on *minimum representations*: known results using the *minimal face* and new results using the *minimal subspace*.

We then study conditions that guarantee a positive duality gap. Necessary and sufficient conditions for a positive duality gap are given, and it is shown how these can be used to generate instances satisfying this property. We then present an algorithm that solves in an efficient and stable way, feasible conic convex optimization problems, including those for which the Slater constraint qualification fails. In addition, we present new relations between strict complementarity and duality gaps.