Power of Duality

Find optimal trajectory/control (for rocket)

$$\begin{aligned} \mu_0 = \min \quad & J(u) = \frac{1}{2}\|u(t)\|^2 = \frac{1}{2}\int_{t_0}^{t_1} u^2(t)dt \\ \text{s.t.} \quad & \dot{x}(t) = A(t)x(t) + b(t)u(t) \\ & x(t_0) = x_0, x(t_1) \geq c. \end{aligned}$$

Using fundamental solution matrix Φ

$$x(t_1) = \Phi(t_1, t_0)x(t_0) + \underbrace{\int_{t_0}^{t_1} \Phi(t_1, t)u(t)b(t)dt}_{\text{integral oper. } \mathcal{K}u}$$

Duality cont...

Convert Pgm $\begin{cases} \min & J(u) = \frac{1}{2}\|u(t)\|^2 \\ \text{s.t.} & \mathcal{K}u \geq d \end{cases}$

Lagrangian dual (best lower bound) is

$$\begin{aligned} \mu_0 &= \max_{\lambda \geq 0} \min_u \{J(u) + \lambda^T(d - \mathcal{K}u)\} \\ &= \max_{\lambda \geq 0} \lambda^T Q \lambda + \lambda^T d \quad \text{simple FIN. dim. QP} \end{aligned}$$

where $Q = -\frac{1}{2}\int_{t_0}^{t_1} \Phi(t_1, t) b(t) b(t)^T \Phi(t_1, t) dt$

$u_*(t) = \lambda_*^T \Phi(t_1, t) b(t)$