A Novel Power Quality Monitoring Allocation Algorithm

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***Abstract*—Distributed measurement architecture, due to its efficiency, is used to monitor the power quality in the electric systems. This paper introduces a novel algorithm to determine the optimum allocations of power quality monitors to reduce the cost of the distributed monitoring system taking into account the data redundancy. The optimization problem is formulated as a covering and packing one which can be manipulated by integer linear programm algorithms. The appropriate constraints are deduced by the electrical circuit topology independent of the load parameters. In addition, the problem is adopted to a general form which can be implemented by any optimization package. The proposed algorithm is applied to five different systems for validation. Then, the results are discussed in terms of the cost and data redundancy.**

***Index Terms*—Covering and packing, deregulation, linear integer programming, power quality, power quality info nodes (PQIN), power quality monitoring, TOMLAB.**

I. INTRODUCTION

WHILE reliability measures the availability of electric service to end-users, power quality (PQ) measures a wide range of power supply characteristics that also can influence the performance of equipment and processes. In other words, the reliability of end-use processes is dependent on both the reliability and quality of the electric service.

Under deregulation, an improved PQ is not only in the interest of the consumer, but it is also beneficial for the supplier/distributor. In the present deregulation era, the customers have the option to buy electricity from various retailers [1], so the customer definitely will choose the supplier who provides the cheapest electricity with acceptable reliability and quality that meets the load needs [2].

Since power quality monitoring (PQM) is the first step in PQ assessment and mitigation, the importance and the complexity of PQM has been recognized [3]. Due to the advances in PQM technologies and algorithms, the recent PQM systems are characterized by flexibility, reliability and speed. Therefore, more utilities are wanting to monitor their systems to evaluate the PQ [1].

Although the PQ can be monitored at a single bus at reasonable cost, the current trend is to move away from monitoring one bus to monitoring the whole system [4]. Such monitoring system can aid in many different applications such as 1) PQ diagnosis, 2) locating the PQ events, 3) sharing information among remote sites, 4) studying the propagation of PQ problems in the system, 5) evaluating the PQ cost effect, and 6) enhancing predictive maintenance programs. So, the new trend in PQ monitoring is the evolution from one separate device into monitoring the whole electric system.

Monitoring a system faces two main problems. The first problem is the communication among the remote sites while the second is the high cost of the monitoring system itself. To solve the first problem, recent advances in Internet communication and real-time applications have been adapted to monitor the entire power system. Reducing the cost associated with the monitoring procedure can be achieved by three different methods: 1) reducing the cost of the PQ monitors, 2) optimizing the number of PQ monitors to be installed, and 3) combining both. Although the first aspect is addressed in a number of publications that try to reduce the cost by using the Distributed Monitoring Scheme [4]–[7], the second and the third aspects have not been thoroughly addressed. This paper concentrates on reducing the cost associated with the monitoring system through the reduction of the number of installed monitors.

In this paper, a novel algorithm is developed that defines the optimum number and locations of the PQ monitors and minimizes the cost of the PQM system, and observe the voltages and the currents at all the buses. The algorithm depends on installing a number of PQ InfoNodes (PQINs) to measure a pre-determined number of currents and voltages, so that the remaining currents and voltages can be calculated. As a result, all the system currents and voltages are observable. The algorithm uses the concepts of covering and packing.

This paper is organized into ten Sections. The distributed PQM and the covering and packing concept are described in Section II and Section III respectively. The mathematical formulation of the problem is presented in Section IV. Both Sections V and VI are dedicated to a description of Ohm's law and Kirchof's law constraints, respectively. Section VII presents the solution algorithm and linearization of the problem. After, the data redundancy is discussed in Section VIII, five study cases are presented in Section IX. Section X concludes the paper.

II. DISTRIBUTED POWER QUALITY MONITORING (DPQM)

To carry out the PQ analysis of a system, the instantaneous waveforms of all the voltages and the currents through out the network should be available [7], [8]. Therefore, PQ monitors should be installed at each bus to measure the currents and the voltages at each bus. Then, the captured data are sent to a server for further analysis [9]. The design of such a system requires the

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analysis of two schemes: the centralized and the decentralized measurement systems. Although the central system is simpler in implementation, it is unreliable due to the dependency on the central unit. Therefore, it is preferable to utilize the decentralized system where, the control is distributed among a number of units and the central unit plays a smaller role [6], [10].

A. Proposed Method

Distributed measurement architecture depends on monitoring some voltages and currents by using a PQIN and on computing the other voltages and currents. The problem of determining the number and the location of measuring devices, which are the PQIN, is known as the observability analysis. To conduct such an analysis, the entire system parameters, the transmission line, transformer, and load parameters, should be known in advance [11]. In an electrical power system, the transmission line and transformer parameters are known; however, the load parameters are not known in advance. It is logical, then, that for the observability analysis of the power system, the circuit topology is relied on to avoid the dependency on the load parameters [11]. In this paper, the optimization problem is formulated in the form of covering and packing, where the density matrix is deduced in relation to the circuit topology.

Although the models accuracy of the power system elements, such as transformers and transmission lines, will not affect the allocation of the PQIN, it has influence on the accuracy of the estimation method. From practical point of view, it is important to monitor the bus voltage either by direct measurement or by estimation with acceptable accuracy level. The voltage at non-measured buses can be accurately estimated using one of the estimation methods that deals with the system uncertainties such as Weighted Least Square method or Kalman's Filter [12], [13]. In general, the models and estimation method shall be chosen adequately to get plenty of accuracy.

Since during and after a short circuit, the circuit topology changes, the observability of the system will change as well. This situations are known as power system observability contingency [13]. Different methods are suggested to deal with this kind of contingency and can be adopted in the proposed method [14], [15].

B. PQIN Description

The PQIN, depicted in Fig. 1, consists of the appropriate transducers, depending on the voltage or current levels, that capture the voltage or the current signal in the analog form. The analog signal is digitalized by a data-acquisition card (DAQ) at a suitable sampling rate and resolution [16], [17]. The components of such a setup are divided into the hardware and software. The hardware consists of: 1) transducers, 2) DAQ card, 3) computers or signal processor setup, and 4) facility to send this data to the server. The software consists of: 1) dynamic state estimation, 2) PQ events detection, classification, location etc., and 3) graphical user interface (GUI).

According to this configuration, the cost of PQIN is defined as

$$C(j) = C_{\text{tran}} + C_{\text{DAQ}} + C_{\text{PC}} + C_{\text{com}} + C_{\text{sw}} \tag{1}$$

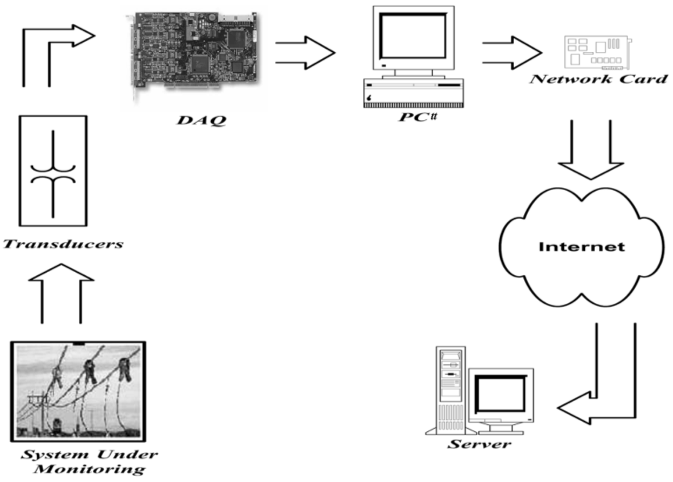

Fig. 1. PQIN configuration.

where C_{tran} is the cost of the transducers, C_{DAQ} is the cost of the DAQ, C_{PC} is the cost of the computer, C_{com} is the cost of the communication Facility and C_{sw} is the cost of the software divided by the number of PQINs.

The cost of the PQIN varies according to the following: 1) the number, sensitivity, and the accuracy of the transducers, 2) the sample rate and the resolution of the DAQ, 3) the communication method, and 4) the cost of the software for the system.

C. Communication Between PQINs

Recently, different power system communication methods and protocols have been used to transfer data and information through the power system. These methods can be wireless, such as satellite, or wired, such as fiber optic, or even renting public communication line [18]. As a result of the advances in Internet communication, it has been more reliable, more secure and cheaper than the aforementioned methods. The use of Internet as a communication environment for both on-line and off-line measurement has been addressed in a number of publication with a great success [6], [19]. By using the appropriate protocol, the real-time synchronization of the data can be accomplished [9], [20], [21]. Therefore, it is suggested to use Internet to communicate among PQINs.

III. COVERING AND PACKING PROBLEM

For a given family of subsets S of a finite set X, the problem of covering is defined as to find a subfamily with the minimum number of members of subsets S such that their union is X [22]. While the problem of packing is to find a subfamily of disjoint subsets that includes the maximum number of members.

The density of covering or packing is the mean number of subsets that covers the elements of the basic set. By definition, the density of any covering is not less than 1, and the density of any packing is not more than 1. The case where the density is equal to 1 is ideal. In this case, the covering is at the same time packing; such objects are usually called perfect.

The problems of covering and packing could be represented by reducing them to problems of Integer Linear Programming (ILP)[22] where X will be the optimization variable, C be the cost of each variable and D be a binary matrix called density matrix. The inner product of two vectors $C = (c_1 \dots c_n)^t$ and $X = (x_1 \dots x_n)^t$ is equal to

$$\sum_{i=1}^{M} c_i x_i = C^t X \tag{2}$$

and constitutes the objective function. The covering problem is defined as minimizing of (2) subjected to $DX \geq 1$, while the packing problem is to minimize (2) subjected to $DX \leq 1$. The columns of the density matrix represent the subsets S and the rows represent the elements of X to be covered. For example, consider $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$ and subsets $S_1 = \{x_1, x_2, x_3, x_4\}$, $S_2 = \{x_4, x_6, x_8, x_9\}$, $S_3 = \{x_3, x_5, x_6, x_8\}$ and $S_4 = \{x_1, x_5, x_7, x_9\}$. The density matrix is given as

$$\mathbf{D} = \left(\begin{array}{c|cccc} \frac{X}{S} & S_1 & S_2 & S_3 & S_4 \\ x_1 & 1 & 0 & 0 & 1 \\ x_2 & 1 & 0 & 0 & 0 \\ x_3 & 1 & 0 & 1 & 0 \\ x_4 & 1 & 1 & 0 & 0 \\ x_5 & 0 & 0 & 1 & 1 \\ x_6 & 0 & 1 & 1 & 0 \\ x_7 & 0 & 0 & 0 & 1 \\ x_8 & 0 & 1 & 1 & 0 \\ x_9 & 0 & 1 & 0 & 1 \end{array} \right). \tag{3}$$

The ILP problem is to find the minimum number of subsets S that cover all elements.

The problem of the allocation of the PQ monitors can be formulated as a covering and packing concept as follows:

> *Given:* the available locations of PQINs are $X = (x_1 \dots x_n)^t$, and the cost of these PQINs is given as: $C = (c_1 \dots c_n)^t$.
> *Problem:* Find the locations of PQINs to minimize the total cost?
> *Constraint:* All the state variables, *i.e.*, the voltages and the currents of the system, must be covered by at least one PQIN "$DX \geq 1$". By saying that PQIN covers a state variable, we mean that the PQIN is capable of observing this state variable either by direct measurement or calculation.

Generally, the density matrix, D, is a binary matrix that depends on the circuit parameters, and requires a prior knowledge of the load parameters. Knowing all the loads' parameters is not a realistic assumption. Therefore, the density matrix needs to be built independently on knowing the load parameters. In this work, the density matrix will be built depending on the power system circuit topology. Generally, the density matrix is the mathematical representation of the constraints.

IV. MATHEMATICAL FORMULATION

Consider an electrical power system of n busses, l lines, and m state variables. Some variables that are used in the mathematical formulation are then defined.

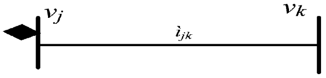

Fig. 2. Part of the power system consists of a transmission line, represented by series inductance and resistance, running between two buses.

Definition 1 "Existence Vector": The existence vector, X, is defined as a vector of n-binary elements which represents the existence of the PQIN and is expressed as

$$x(j) = \begin{cases} 1, & \text{if PQIN \# } j \text{ is installed} \\ 0, & \text{if PQIN \# } j \text{ is not installed.} \end{cases} \tag{4}$$

Definition 2 "Cost Vector": The cost vector, C, is defined as a vector of n-elements that expresses the cost of each PQIN, where

$$c(j) = \text{the cost of PQIN}\#j. \tag{5}$$

The total cost of the monitoring system is the summation of the cost of the installed monitors and is given in (2). This objective function should be minimized subjected to the observability constraints which are deduced by applying Kirchof's Current Law (KCL) and Ohms's Law (OL) resulting in two groups of constraints.

V. OHM'S LAW CONSTRAINTS

Fig. 2 shows two buses connected by a transmission line. By applying Ohm's law to the circuit, the relation between the current and the voltages can be written in the time domain (6)

$$v_j - v_k = R i_{jk} + L \frac{di_{jk}}{dt}. \tag{6}$$

Since there are three state variables: i_{jk}, v_j,and v_k, in this equation, knowing two variables leads to the calculation of the third variable. From this equation, the following lemmas are true.

Lemma 1: If the voltage at one bus of the line and the current through it are observable, then the voltage at the other bus is observable.

Lemma 2: If the voltages across the line are observable, then the current through this line is observable.

These two lemmas form two constraints: voltage constraints and current constraints, respectively.

A. Voltage Constraints

Definition 3 "Connectivity Matrix": The connectivity matrix, A, is defined as a binary $(m \times n)$-matrix with column $\#k$ representing the PQIN at bus $\#k$ and the row v_j representing the state variable v_j (the voltage at bus $\#j$). The elements of this matrix are defined as

$$a(v_j, k) = \begin{cases} 1, & \text{if } v_j \text{ is observed by PQIN } \#k \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Note that; $A(v_k)$ points to the row corresponding to the state variable v_k and $A(j)$ points to column corresponding to the PQIN$\#j$ at bus number j.

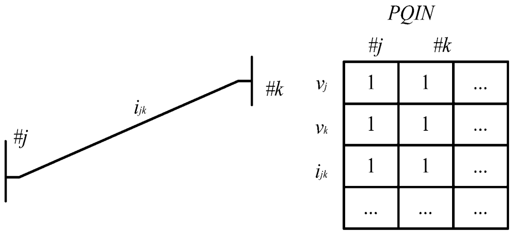

Fig. 3. Stamp of matrix A for a transmission line.

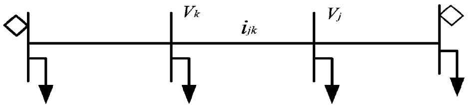

Fig. 4. Four buses are connected by lines as a part of the power system.

From the definition of A, the condition of observation of v_j is the installation of $\text{PQIN}\#k$, $x(k) = 1$, and the ability of $\text{PQIN}\#k$ to observe v_j, $a(v_j, k) = 1$, in other words, $a(v_j, k)x(k) = 1$. Since the goal is to observe v_j by at least one PQIN, then the following condition should be valid $a(v_j, 1)x(1) + a(v_j, 2)x(2) + \cdots a(v_j, n)x(n) \geq 1$. In other words

$$\sum_{k=1}^{n} a(v_j, k)x(k) \geq 1. \tag{8}$$

Definition 4 "Observability Vector": The observability vector U, is defined as an integer vector of n-elements, representing how many times the state variable is observed, and is

$$U = AX. \tag{9}$$

Note that $u(k) = T$ indicates that the state variable k is observable at T times, where $T = 1$ is the ideal case. However, some state variables can be observed by more than one PQIN and data redundancy can occur.

The condition of observing all the state variables is that all the elements in vector U must equal to at least one. Since the observability vector, U, depends on the locations of the PQINs, the vector X, and the circuit topology of the transmission line, matrix A, the system observability is built without knowing the load parameters.

As mentioned before, the task of a PQIN is to measure voltages and currents at the installed bus. Since, both the voltage and the current are known, the voltage at a connected bus is observed according to lemma 1. Therefore, a PQIN at any bus will be able to observe the voltage at all the buses connected to it.

For a line running between bus $\#j$ and bus $\#k$, the state variables are v_j, v_k and i_{jk}. It is obvious that installing $\text{PQIN}\#j$ yields $a(v_j, j) = a(v_k, j) = a(i_{jk}, j) = 1$ whereas installing $\text{PQIN}\#k$ yields $a(v_j, k) = a(v_k, k) = a(i_{jk}, k) = 1$. Accordingly, the stamp of A-matrix for a transmission line is shown in Fig. 3.

B. Current Constraints

Consider Fig. 4 where a part of a power system consists of four buses and two PQINs is illustrated.

If the two PQINs are installed as shown, by a rhombus, then the voltages (v_j, v_k) will be observed by these two PQINs, according to lemma 1; and consequently the current through this line, i_{jk}, is observable, according to lemma 2,. Typically, the following is true

If v_j and v_k are observable, then i_{jk} is observable

Since the U-vector is the observability vector, then

If $u(v_j) \geq 1$ and $u(v_k) \geq 1$ then $u(i_{jk}) \geq 1$.

To formulate this constraint mathematically, a definition of the co-connectivity matrix is needed.

Definition 5 "Co—Connectivity Matrix": The co-connectivity matrix B is defined as a binary $(m \times n)$-matrix that is expressed as

$$B(i_{jk}) = \begin{cases} A(v_k), & \text{if bus } \#j \text{ and } \#k \text{ are connected} \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

With the last constraint, the formulation is

If $B_j x \geq 1$ and $B_k x \geq 1$, then $u(i_{jk}) = 1$.

By replacing the *and*-operator by the multiplication-operator

If $(B_j.x)^t \cdot (B_k.x) \geq 1$, then $u(i_{jk}) = 1$.

This equation defines the co-observability vector.

Definition 6 "Co-Observability Vector": The co-observability vector W, is defined as an integer vector if n-elements expressed as

$$w(i_{jk}) = (B_j \cdot x)^t \cdot (B_k \cdot x) \quad \forall\, i_{jk}. \tag{11}$$

For a line carrying current i_{jk} between bus $\#j$ and $\#k$, B-matrices are built according to lemma 2 as follows:

1) Read the data file.
2) Build A-matrix.
3) Set $B_j(i_{jk}) = A(v_j)$ and $B_k(i_{jk}) = A(v_k)$.

The flowchart in Fig. 5 summarizes the steps of building the A and B matrices.

VI. KIRCHOF'S CURRENT LAW CONSTRAINTS

For the previous constraints, it is assumed that the loads at all the buses are unknown. However, enough information about some loads may be available, so that the allocation problem can benefit from this information. The buses whose load parameters are known are called the known buses, whereas the bus that has no loads connected to it is called the connecting bus. These buses should be introduced in the allocation problem to reduce the number of PQINs. To introduce these two buses into the optimization problem, as constraints, KCL will be used.

KCL states that "the incoming and the outgoing currents at any bus are equal". By applying this law on these two types of buses, the result is that there are two extra constraints that reduce the number of the installed PQINs.

A. Known Busbar

Definition 6 "Known Bus": The known bus is defined as the bus whose relation between its load's current and voltage is known; *i.e.*, $f(v, i) = 0$ is known.

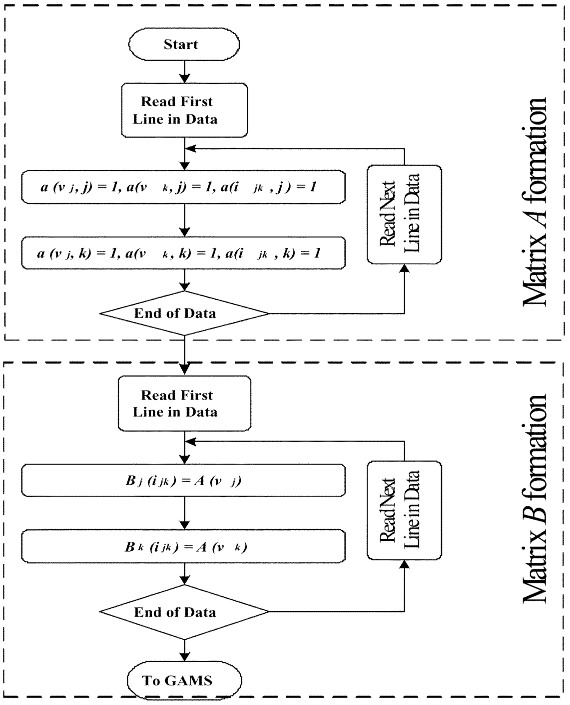

Fig. 5. Flowchart of generating the A and B matrices.

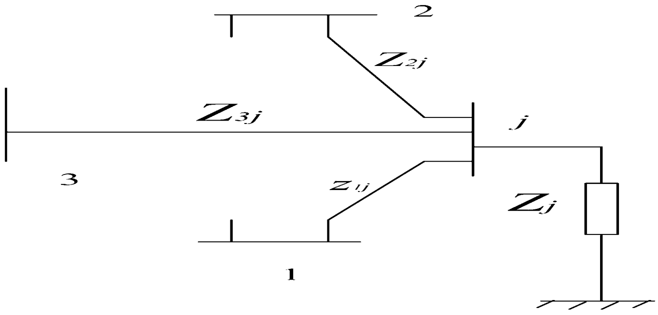

Fig. 6. Known bus. The load is represented by the impedance to indicate that $f(v, i) = 0$ is known.

Consider a part of the power system in Fig. 6, with KCL applied at bus $\#j$ such that

$$i_{j1} + i_{j2} + i_{j3} = i_j \tag{12}$$

if the transmission line and the load impedance are known, then

$$\frac{v_1}{z_{1j}} + \frac{v_2}{z_{2j}} + \frac{v_3}{z_{3j}} = \frac{v_j}{z_{jj}}$$

where

$$\frac{1}{z_{jj}} = \frac{1}{z_j} + \frac{1}{z_{1j}} + \frac{1}{z_{2j}} + \frac{1}{z_{3j}} \dots. \tag{13}$$

Therefore, if v_1, v_2, and v_3 are observable, and $f(v_j, i_j) = 0$ is known, then v_j is observable. In this example, we assume that the load is represented by an impedance z_j, that is

$$f(v_j, i_j) = v_j - z_j \cdot i_j = 0. \tag{14}$$

As a result, the following lemma is valid.

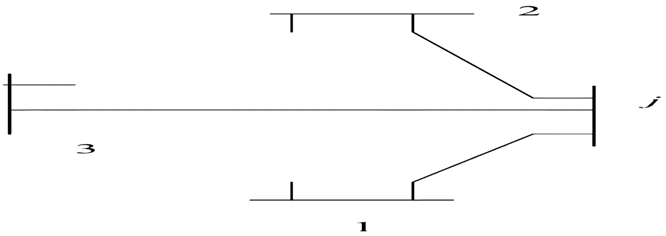

Fig. 7. Connecting bus with no generation unit and no load.

Lemma 3: If all the voltages of the buses, connected to a known bus, are observable, then the voltage at the known busbar is observable.

By applying this lemma in the circuit in Fig. 6, we can say that

3) If $u(1) \geq 1$ *and* $u(2) \geq 1$ *and* $\dots u(J) \geq 1$, then v_j is observable

By replacing the *and*-operator by the multiplication-operator, the observability condition of v_j is

$$u(1) \cdot u(2) \dots u(J) \geq 1. \tag{15}$$

By replacing u in (8)

$$\prod_J \sum_{k=1}^{n} a(j,k) \cdot x_k \geq 1. \tag{16}$$

This condition leads to a definition of the KCL co-observability vector

Definition 7 "KCL Co—Observability Vector": The co-observability vector y, is defined as an integer vector of n-elements, expressed

$$y(v_j) = \prod_J \sum_{k=1}^{n} a(j,k) \cdot x_k. \tag{17}$$

The Y-vector is written for only known buses and it depends on knowing the load parameters in advance. It is not worthy that the more that is known about the buses, the fewer PQINs are required.

B. Connecting Busbar

Define the connecting bus as the bus that is connected to neither the load nor the generator. Fig. 7 shows a part of the power system, where bus $\#j$ is called the connecting busbar. This constraint is considered as a special case of the known bus. The reason for considering it to be a different constraint is that it requires different treatment in the estimation method. The mathematical formulation is the same as in (17).

VII. SOLUTION ALGORITHM

The optimization problem can be stated as

$$\begin{cases} \min \sum_{j=1}^{n} c_j \cdot x_j \\ \text{subjected to} : U + W + Y \geq 1. \end{cases} \tag{18}$$

As seen from the previous sections, X is a binary vector so that there is a nonlinear optimization problem caused by the vectors W and Y. To overcome the nonlinearity caused by W, we restate the constraints as

$$\begin{cases} U + W_j + Y \geq 1 \\ U + W_k + Y \geq 1 \\ \text{where}: W_j = B_j \cdot X \text{ and } W_k = B_k \cdot X. \end{cases} \tag{19}$$

The same concept can be applied to linearize the vector y. Solving the problem of covering and packing by the ILP is beyond the scope of this paper, and more details can be found in [22]. In this paper, it is adequate to formulate the problem and solve the ILP problem by utilizing the Branch and Bound method [23]. During the formulation of the equation, the mathematical equations are formulated in a general form that can be manipulated by any general purpose optimization software. For a large electrical system, TOMLAB is used as the optimization software since TOMLAB has proven its efficiency with dealing in large scale optimization problems [23], [24].

The last point to be considered here is related to the cost of the PQIN. In some systems, there will be PQIN installed already to monitor either the bus or the transformer; therefore, the cost due to such a PQIN is zero. However, the cost of such a PQIN should be set to a very small value in the optimization cost function (18); otherwise, the optimization will consider the cost of this PQIN as a dummy variable.

VIII. DATA REDUNDANCY

It is always anticipated in monitoring problems that there is data redundancy, which means that some of the network voltages or currents can be measured or calculated from two or more different monitors. Yet, it is desirable to limit this redundancy. In this context, the data redundancy is defined as: how many times the state variables are measured or calculated, and the data redundancy factor (DRF) is defined as

$$\text{DRF} = \frac{\text{sum of numbers of observing state variables}}{\text{number of state variables}}. \tag{20}$$

When there is no redundancy in the data, the DRF is one, and the higher the DRF, the higher the redundancy is.

Equation (18) shows that $u(j)+w(j)+y(j)=T$, indicating that the state variable j is observed T times. So, DRF could be written in terms of u, w, and y as

$$\text{DRF} = \frac{\sum_{j=1}^{N} u(j) + w(j) + y(j)}{N} \tag{21}$$

where

N number of state variables.

The main concern with the optimization problem is to optimize the number of PQINs, and consequently, the cost of the monitoring system. Moreover, different PQIN allocation configurations have the same value as a cost function. However, the DRF differs from one configuration to another. Therefore, the criteria of choosing between those configurations will rely on the data redundancy. The benefits of less data redundancy are the reduction of the bandwidth of sending and receiving the data between the different PQINs as well as reducing the media to store these data. However, reducing the redundancy in the data reduces the system reliability.

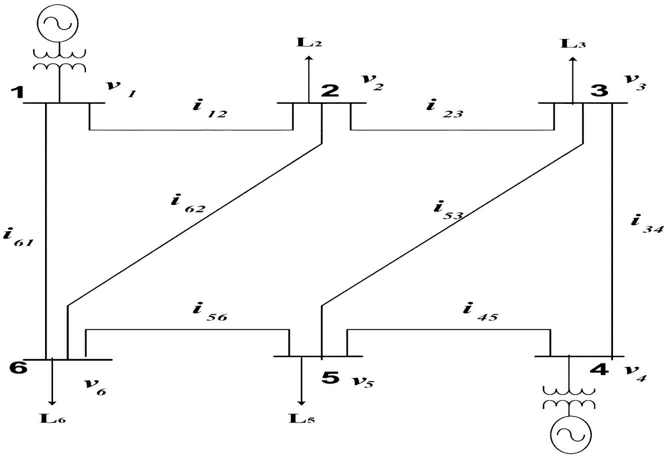

Fig. 8. Six-bus system.

IX. APPLICATIONS

The proposed method is applied to different systems, where TOMLAB is used as the optimization package. The first study case is a simple 6-bus system to illustrate the proposed method. The second study case reveals the effect of the known and connecting buses. The effect of data redundancy and the criteria of choosing between the different configurations is illustrated by the third study case. Here, a more realistic system is studied in which the emphasis is on the effect of the number of transmission lines on the PQIN allocation. It should be pointed out here that, whenever series device is installed in the system between buses $\#j$ and $\#k$, it is dealt as a load connected at both bus $\#j$ and bus $\#k$. However, if the dynamic model of the series device is known, it can be dealt as in the transmission-line case.

A. Study Case (1)

A six-bus system with eight transmission lines is shown in Fig. 8. The state variables are the bus voltages and the transmission lines currents as indicated in the figure. In this case, it is assumed that no information is available about either the loads or the generation units (i.e., there is neither known busbars nor connecting busbars). The A and B matrices are built and the system is solved by TOMLAB.

The Appendix details the formulation of A-matrix and B-matrix. Although the cost of the PQIN at each bus depends on the number and the types of the sensors as well as the bus location, equal cost for all PQINs is assumed in this study case for simplicity. The optimum number of monitors is found to be 2 PQIN. The different locations of the PQIN and the associated DRF are given in Table I.

In this allocation, #1 and #4, it is obvious that PQIN#1 will measure v_1, i_{12}, and i_{61}, and the voltages v_2 and v_6 will be calculated by knowing the line resistance and inductance. Also,

TABLE I
OPTIMUM LOCATIONS AND DRF OF THE SIX-BUS SYSTEM

PQIN Location	DRF
$\#1\ and\ \#3$	1.57
$\#1\ and\ \#4$	1.28
$\#1\ and\ \#5$	1.5
$\#2\ and\ \#4$	1.57
$\#2\ and\ \#5$	1.78
$\#3\ and\ \#6$	1.78
$\#4\ and\ \#6$	1.5

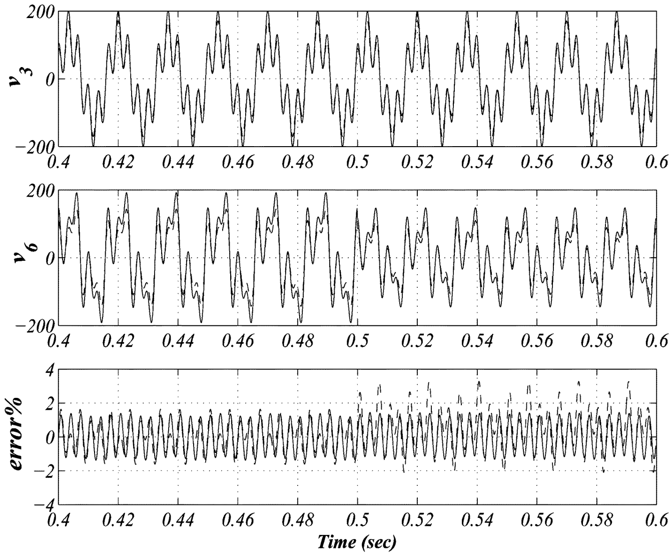

Fig. 9. Voltage at bus #3 and #6 and error.

PQIN#4 will measure v_4, i_{34}, i_{45}, and v_3 and v_5 will be calculated. The current in lines between busbars 2 and 6, i_{62}, will be calculated, as v_2 and v_6 are observable and the line parameters are known. The same concept will be applied for currents i_{23}, i_{53}, and i_{56}. Having all transmission-line currents observable, the load current at all busbars will be observable.

To verify the ability of the two nodes to observe the system accurately, the system is simulated using EMTDC/PSCAD. The system is supplied by two source connect at buses #1 and #4. To simulate different PQ problems, nonlinear loads are connected at buses #3 and #6. The nonlinear loads are simulated by harmonic current sources with the third, fifth, and seventh harmonic. Moreover, a sudden large load is connected at bus #5 after 0.5 s to simulate a voltage sag. The linear loads are connected at buses #2 and #5. The voltages at buses #3 and #6 are given as

$$\begin{cases} v_3 = \left(L_{34}\frac{d}{dt} + R_{34}\right) i_{34} + v_4 \\ v_6 = \left(L_{61}\frac{d}{dt} + R_{61}\right) i_{61} + v_1. \end{cases} \tag{22}$$

By substituting (d/dt) by Euler backward approximation

$$\begin{cases} v_3(t) = L_{34}\frac{i_{34}(t)-i_{34}(t-1)}{T_s} + R_{34}i_{34}(t) + v_4(t) \\ v_6(t) = L_{61}\frac{i_{61}(t)-i_{61}(t-1)}{T_s} + R_{61}i_{61}(t) + v_1(t) \\ \text{where : } T_s \text{ is sampling time.} \end{cases} \tag{23}$$

The voltages at buses #3 and #6, v_3 and v_6, are calculated according to (23) and compared to the actual values from the simulation. Fig. 9 displays the actual waveforms of v_3 and v_6 which are distorted due to the existence of nonlinear loads with a voltage sag in v_6. The error is found to be less than 2% as shown in the figure.

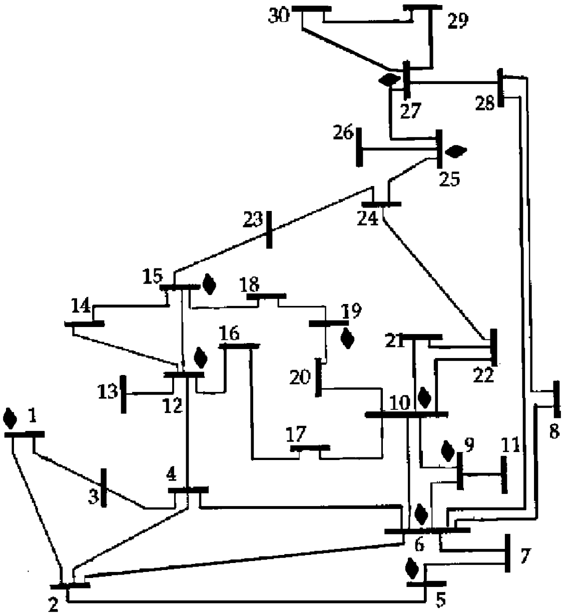

Fig. 10. IEEE 30-bus system with the location of the PQIN indicated by the rhombus in case of equal PQIN cost.

B. Study Case (2)

To show the effect of the KCL constraints, the second study case will be utilizing the same six-bus system, but assuming that buses #5 and #4 are known busbars. A and B matrices will not differ from the previous case and two extra constraints will be added, namely the KCL constraint, at busbars #4 and #5. The optimum number of PQIN is found to be 1 at busbar #2. This study case shows that known busbars will reduce the number of required PQINs.

It is clear that PQIN#2 will measure v_2, i_{12}, i_{23}, and i_{26} and the voltages v_1, v_3, and v_6 will be calculated by knowing the line impedance. To calculate the voltages at bus #4 and #5, consider KCL at busbar #4, we can write

$$\frac{v_3}{z_{34}} + \frac{v_5}{z_{45}} = \frac{v_4}{z_{44}} + f_g(v_4) \tag{24}$$

where f_g is the dynamic function that models the dynamics of the generator and $z_{ij} = R_{ij} + L_{ij}(d/dt)$.

KCL at busbar #5 gives

$$\frac{v_3}{z_{35}} + \frac{v_4}{z_{45}} + \frac{v_6}{z_{56}} = \frac{v_5}{z_{55}}. \tag{25}$$

Since v_3 and v_6 are observable, then we can solve these two equations (24) and (25), to obtain the two unknowns voltages, v_4 and v_5.

C. Study Case (3)

For a more realistic study, the proposed algorithm is applied to the IEEE 30-bus. It is assumed that there is no information

TABLE II
OPTIMUM LOCATIONS AND DRF OF THE IEEE 30-BUS SYSTEM

bus	#1	#2	#3	#4	#5	#6	#7	#8
#1	1	0	1	0	1	0	1	0
#2	1	1	0	0	1	0	1	1
#3	0	0	0	1	0	1	0	0
#4	0	1	0	0	0	0	0	1
#5	0	0	1	1	0	1	0	0
#6	1	1	0	0	1	0	1	1
#7	0	0	0	0	0	0	0	0
#8	0	0	0	0	0	0	0	0
#9	1	1	1	1	1	1	0	1
#10	1	1	1	1	1	1	1	1
#11	0	0	0	0	0	0	1	0
#12	1	1	1	1	1	1	1	1
#13	0	0	0	0	0	0	0	0
#14	0	0	0	0	0	0	0	0
#15	0	0	0	0	1	1	0	0
#16	0	0	0	0	0	0	0	0
#17	0	0	0	0	0	0	0	0
#18	1	1	1	1	0	0	1	1
#19	0	0	0	0	0	0	0	0
#20	0	0	0	1	1	1	0	0
#21	0	0	0	0	0	0	0	0
#22	0	0	0	0	0	0	0	0
#23	0	0	0	0	0	0	0	0
#24	1	1	1	1	0	0	1	1
#25	1	1	1	1	1	1	1	1
#26	0	0	0	0	0	0	0	0
#27	1	1	1	0	0	0	1	0
#28	0	0	1	1	0	1	0	0
#29	0	0	0	0	1	1	0	1
#30	0	0	0	0	0	0	0	0
DRF	1.97	2.12	1.64	1.64	2.01	1.67	1.81	2.15

about either the generation units or the loads. It is assumed here that all the PQINs have equal costs. The results are shown in Fig. 10, where the PQINs are indicated by the rhombus. In this system, it is noticed that the redundancy factor DRF is a bit higher than the one in the previous study due to the fact of heavy connectivity of the system. However, removing any of these PQINs will result in loosing the observability of another state variable.

As mentioned earlier, in some cases a higher DRF is needed to obtain a higher reliability. Consequently, it is required to list all the locations of the PQIN that have the same cost function value, then choose the highest or the lowest DRF depending on the required monitoring reliability. Table II indicates different locations of the PQINs with the associated DRF. Although the optimum PQIN's number of all the configurations is ten PQINs, the DRF is found to vary between 1.643 and 2.155.

Since the cost of the PQIN is not equal at all of the buses, it is beneficial to study the case of unequal PQINs cost. For that sake, the PQIN cost in the system under study will vary and the optimization process will be conducted to determine the optimum configuration that gives minimum overall cost.

Equation (1) shows that the cost of the PQIN is a function of different factors where the number of transducers plays the major role in the cost. More transducers means: 1) wider communication bandwidth is needed which increases the communication cost (C_{com}), 2) more DAQ analog channels are required which increases the cost of the DAQ (C_{DAQ}), and 3) escalating the transducers cost it self (C_{tran}). Therefore, the PQIN cost is chosen to proportionate with the number of transducers, in other words the PQIN cost is proportional to the number of connected lines to that bus.

TABLE III
IEEE 118 AND 300-BUS SYSTEMS

System	No. of Lines	No. of PQIN	%Saving	DRF
IEEE-118	186	32	72.88%	1.45
IEEE-300	411	41	86.33%	1.52

Based on the aforementioned discussion, the PQIN cost of the IEEE 30-bus system has been chosen based on the number of transducers. For example, bus #2 needs three potential transformers to measure the three-phase voltages and fifteen current transformers to measure the currents of the four lines and the load connected at the bus. As a result, the $c(2)$ is set be 18 in (4).

The optimum number is found to be twelve-PQIN at buses number 3, 5, 8, 11, 13, 14, 17, 19, 21, 23, 26, and 29. Although, the number of the PQIN has been increased from ten to twelve compared to the case of equal cost, the actual cost of the chosen twelve is less than the ten.

D. *Study Case (4)*

In this case, larger systems are tested to study the effect of the circuit topology. The first system is the IEEE-118 bus system with 186 connecting lines. Due to the fact that the system is heavily connected, the optimum number of the PQIN is found to be 32 out of 118 available locations, so saving of 72.88% is achieved.

The second case is the IEEE 300-buses system which has 411 connection lines. The optimum number of the PQIN is found to be 41. Table III summarizes these results.

X. CONCLUSION

This paper introduces a novel algorithm to determine the optimum number and location of power quality monitors while keeping the system state variables, currents and bus voltages, observable. The problem is formulated in covering and packing concept, and is transferred to the problem of integer linear programming which can be manipulated by any general purpose optimization program, such as TOMLAB. Introducing this technique will solve the high cost problem associated with monitoring the entire electrical power system.

The proposed algorithm is applied to a simple 6-bus system and has reduced the number of PQIN by 66%. Moreover, utilizing the concept of known buses results in a further reduction of installed monitors and increased the savings to 83%.

However, applying the proposed algorithm to the IEEE-30 bus system results in a saving of 66.6%, and a number of solutions have the same minimum cost. Therefore, a data redundancy factor is used in choosing among the solutions.

With heavily connected systems such as IEEE-118 and IEEE-300 systems, the saving is increased to 72.8% and 86.3%, respectively. Therefore, the amount of saving is fairly depending on the topology of the power system connections.

Appendix
Connectivity and Co-Connectivity Matrices of Case (1)

$$\mathbf{A} = \left(\begin{array}{c|cccccc}
\frac{PQIN}{SV} & \#1 & \#2 & \#3 & \#4 & \#5 & \#6 \\
\hline
v_1 & 1 & 1 & 0 & 0 & 0 & 1 \\
v_2 & 1 & 1 & 1 & 0 & 0 & 1 \\
v_3 & 0 & 1 & 1 & 1 & 1 & 0 \\
v_4 & 0 & 0 & 1 & 1 & 1 & 0 \\
v_5 & 0 & 0 & 1 & 1 & 1 & 1 \\
v_6 & 1 & 1 & 0 & 0 & 1 & 1 \\
i_{12} & 1 & 1 & 0 & 0 & 0 & 0 \\
i_{23} & 0 & 1 & 1 & 0 & 0 & 0 \\
i_{34} & 0 & 0 & 1 & 1 & 0 & 0 \\
i_{45} & 0 & 0 & 0 & 1 & 1 & 0 \\
i_{56} & 0 & 0 & 0 & 0 & 1 & 1 \\
i_{61} & 1 & 0 & 0 & 0 & 0 & 1 \\
i_{62} & 0 & 1 & 0 & 0 & 0 & 1 \\
i_{53} & 0 & 0 & 1 & 0 & 1 & 0
\end{array}\right) \tag{26}$$

$$\mathbf{B_j} = \left(\begin{array}{c|cccccc}
\frac{PQIN}{SV} & \#1 & \#2 & \#3 & \#4 & \#5 & \#6 \\
\hline
v_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
i_{12} & 1 & 1 & 0 & 0 & 0 & 1 \\
i_{23} & 1 & 1 & 1 & 0 & 0 & 1 \\
i_{34} & 0 & 1 & 1 & 1 & 1 & 0 \\
i_{45} & 0 & 0 & 1 & 1 & 1 & 0 \\
i_{56} & 0 & 0 & 1 & 1 & 1 & 1 \\
i_{61} & 1 & 1 & 0 & 0 & 1 & 1 \\
i_{62} & 1 & 1 & 0 & 0 & 1 & 1 \\
i_{53} & 0 & 0 & 1 & 1 & 1 & 1
\end{array}\right) \tag{27}$$

$$\mathbf{B_k} = \left(\begin{array}{c|cccccc}
\frac{PQIN}{SV} & \#1 & \#2 & \#3 & \#4 & \#5 & \#6 \\
\hline
v_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
i_{12} & 1 & 1 & 1 & 0 & 0 & 1 \\
i_{23} & 0 & 1 & 1 & 1 & 1 & 0 \\
i_{34} & 0 & 0 & 1 & 1 & 1 & 0 \\
i_{45} & 0 & 0 & 1 & 1 & 1 & 1 \\
i_{56} & 1 & 1 & 0 & 0 & 1 & 1 \\
i_{61} & 1 & 1 & 0 & 0 & 0 & 1 \\
i_{62} & 1 & 1 & 1 & 0 & 0 & 1 \\
i_{53} & 0 & 1 & 1 & 1 & 1 & 0
\end{array}\right) \tag{28}$$

$SV =$ State Variables.

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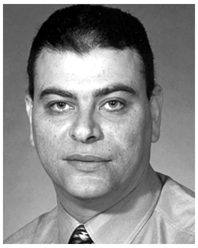

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