Optimal acquisition policy with quantity discounts and uncertain demands

Guoqing Zhang[a,*], Liping Ma[b]
[a]Department of Industrial and Manufacturing Systems Engineering
University of Windsor,
Windsor, ON, Canada N9B 3P4
E-mail: gzhang@uwindsor.ca
[b]Aspen Technology Canada
Calgary, AB, Canada T2G 0P6
E-mail: lipingma@gmail.com

[*]Corresponding author (Tel: (519)253-3000 Ext. 2637, Fax: (519) 973-7062).

Abstract

We study the acquisition policy decision problem for a supply network involving one manufacturer and multiple suppliers. The manufacturer produces multiple products under uncertain demands and each supplier provides price discounts. The problem is to determine the manufacturer's acquisition policy and production levels so as to maximize the manufacturer's expected profit, subject to both the manufacturer's and suppliers' capacities. We present a mixed integer nonlinear programming (MINLP) formulation of the problem, for both single- and multiple-sourcing procurement policies. GAMS and its solvers, combining external integration functions, are employed to solve the complex MINLP problem. The preliminary computation results and managerial analysis are reported.

Keywords: supplier selection, uncertainty, quantity discount, mixed-integer nonlinear programming

1 Introduction

Most manufacturers are under a continuous pressure to decrease cost. Reducing the outsourcing cost is one of the primary determinants of profitability. For example, the range of purchasing's share in the total turnover of industrial firms typically is between 50-90% (Boer *et al.*, 2001). Supplier selection is a crucial acquisition policy that has long-term impact on outsourcing cost.

The selection of suppliers is a decision problem to select suitable vendors and allocate orders to different vendors with the major objective of reducing the expense of purchasing raw material and component parts. The problem has been studied extensively since the 1960s and attracted more attention in recent years as the major issue of supply chain design. The vast majority of the research on the field has assumed the product demands are deterministic and known. However, it is pervasive in practice that the firms face uncertain demands in their product/service markets, which increases the complexity of their decisions and risk. Therefore, taking into account uncertain demands in supplier selection is a crucial subject, both from research perspective as well as from a practical point of view.

Kim *et al.* (2002) presented a nonlinear programming (NLP) model to configure a supply network with uncertain demand. This paper extends their work to a new environment where both uncertain demands and quantity discounts are present. In details this study includes the following main features. First, we take into account demand uncertainty in the supplier selection decision. One difficulty caused by demand uncertainty is that the production level depends on product demands and is unknown, while the production level of each product is a key element to determine how many raw materials/parts to purchase. Therefore we develop an integrated optimization model to determine production planning, and supplier selection sententiously, subjected to both manufacturer and suppliers capacity constraints. Second, our problem includes the circumstance that suppliers provide different quantity discount schemes. Quantity discount is a very common approach employed by suppliers to promote their products. Given a quantity discount, the manufacturer can procure raw material at a lower unit price if the order or contract quantity is over a certain value - a price break point. However, it will increase over-order and inventory risk, especially when the demand is uncertain. The supplier selection problem under uncertain environment becomes more challenging when quantity discount schemes are present. Other features include both single- and multiple-sourcing policies, and supplier management costs (explained later in Section 3) are considered in our models.

To the best of our knowledge, it is the first investigation in the literature that explores the impact of introducing both uncertain demands and price discounts on supply selection and production planning. Our objectives are to develop the optimal acquisition policy for supply selection in this context and to gain insight into the factors affecting the manufacturer profit.

We present a mixed integer nonlinear programming (MINLP) formulation of the problem, with the objective of maximizing the manufacturer's expected profit. A variant of the model can be used to find manufacturer's optimal capacity in the circumstance, one of the strategic decision problems the manufacturer faces. Given the complexity of the MINLP model, closed form expressions for the optimal solution could not be obtained. Instead, we employ GAMS and its solvers, combining with external integration functions, to solve the problem. To our knowledge, it is one of very few works reported to use GAMS and its solvers to solve MINLP problems.

An outline for this paper is as follows. Section 2 provides the brief literature review of procurement with uncertainty or price discount. Section 3 presents MINLP models. Section 4 provides a solution approach. Computing test and managerial analysis are described in Section 5. An extension to volume discounts is discussed in Section 6. We finally conclude the paper in Section 7.

2 Literature Review

To link with the background of this research, we mainly review the studies on the vendor selection under price discount or uncertainty environments, by comparing with our problem and model. (For other reviews see Boer *et al.* (2001) and Aissaoui *et al.* (2006).)

2.1 Vendor selection under price discount environments

Price discount is a common and effective policy for vendors to promote their products. Traditionally, vendors offer two types of price discount schemes: quantity discount and volume discount. The former is based on the quantity of an item purchased - promoting the buyer to order large quantities of a given item; the latter is based on the total dollar value of all items purchased from one supplier, usually to promote more balanced sales over multiple products. From the methodology point of view, mixed integer programming (MIP) models, nonlinear programming models and Economic Order Quantity (EOQ) formula are the main approaches employed to describe these vendor selection problems where vendors have discount schedules. Different from our model, the majority of the research on supplier selection has assumed that the product demands are deterministic and known. Excepted as noted, those models did not take into account demand uncertainty.

Chaudhry *et al.* (1993) studied the vendor selection problem where the selection process was influenced by the price, delivery, and quality objectives of the buyer, and the vendors offered price breaks. Mixed integer linear programming models, not MINLP, are presented for both cumulative and noncumulative price breaks. The problem assumes there is a single product and the total order quantity is known. Parlar and Wang (1994) presented a mathematical model that analyzes how to set quantity discounts so that the buyer and seller simultaneously gain maximum profitability. The model was developed based on the assumptions of the EOQ model and there are no lost sales or backorders.

Sadrian and Yoon (1992) compared the practical impact of the volume discount with that of the quantity discount and concluded that the joint volume discount is more realistic for both vendors and buyers. Sadrian and Yoon (1994) extended their work with a mixed integer (0-1) programming model and developed a procurement decision support system. The system combined both annual commitment and as-ordered purchasing policies with the assumption of given forecast demands. Xu *et al.* (2000) developed a mixed integer programming model for a multi-item lot size problem with a volume discount schedule. Rather than using the mathematical programming software, a heuristic approach based on dynamic programming was developed to solve the model more quickly. The model is deterministic in nature, and no back order is considered. Dahel (2004) presented an MIP approach to determine the number of vendors and order allocation to these vendors in multiple-product, volume discount environments. The model has multiple objectives, including the price, delivery, and quality, subject to the capacity constraints of the vendors.

A third type of discount scheme, total quantity discounts, has appeared in recent years. This is a combined policy of quantity discount and volume discount: the discount is based on the total quantity of multiple items purchased instead of total dollar value. Crama *et al.* (2004) described a tactical purchasing planning problem for a multi-plant company, where the suppliers offer discounts based on the total quantity of ingredients purchased. The purchasing decisions are also effected by alternative product recipes for each final product. Several nonlinear mixed integer programming models are presented to formulate the problem and then linearized. However, their models do not take into account any capacity constraints and demand uncertainty.

2.2 Acquisition policy with uncertain demands

Although many studies have been conducted to deal with uncertainty issues in production planning/optimum order quantity context, very few have addressed how to optimize manufacturer's long-term acquisition policy under uncertain demands while taking both manufacturer's and suppliers' capacities into account.

The classical newsboy model has been widely used to determine the optimum single-product order quantity under uncertain demand. Several extensions to the newsboy problem with the consideration of either single or multiple capacity constraints have been presented in the last twenty years, such as, Nahmias and Schmidt (1984), Lau and Lau (1996), and Erlebracher (2000). Different iterative algorithms, mostly based on Lagrangian relaxation, are proposed to solve the problems. However, those models are limited to determine order quantities, without taking into account supplier selection decision.

Kim *et al.* (2002) investigated how to configure a supply network that consists of a manufacturer making multiple products with uncertain consumer demands, with multiple suppliers providing the raw materials/components required for said products' fabrication. A NLP model was presented for this supply network that attempts to find the optimal acquisition policy and the production levels for the manufacturer - maximizing the profit. They developed an iterative analytical algorithm to solve the model using the optimality analysis. But their model does not consider vendors' discount schedules and management cost.

Alonso-Ayuso *et al.* (2003) presented a two-stage stochastic 0-1 modeling and a related algorithmic approach to determine the production topology, plant sizing, product selection, product allocation among plants and vendor selection for raw materials. However, the uncertainty on the stochastic parameters is represented via a set of scenarios, and a discount scheme from suppliers is not included.

Liao and Rittscher (2007) presented a multi-objective supplier selection model under stochastic demand conditions. The computation results with a genetic algorithm are reported. However, the model involves single product and does not take into account manufacturer's capacity and price discount scheme.

The model presented in this paper extends Kim *et al.*'s (2002) to consider the new features of suppliers that provide quantity discount schemes. Additionally, single- and multiple-sourcing policies are also introduced. The resulted model is MINLP with expected objective function instead of NLP, where the presence of an integration function in the objective function makes solving difficult. A solution approach is developed by exploring the current commercial mathematical programming packages.

3 MINLP Models for optimizing acquisition policy

The supply network shown in Figure 1 depicts a manufacturer with multiple products having uncertain product demands, and multiple suppliers providing different combinations of raw materials with quantity discount offers. The challenge becomes finding both a long-term acquisition policy and the optimal production level for the manufacturer that maximizes its expected profit over the assumed production period, subject to both the manufacturer's and suppliers' capacities. The above problem is formulated as MINLP models with the following assumptions:

[Insert figure 1 about here]

- The problem considered here is to determine the long-term production level and related strategic acquisition policy. Inventory cost and ordering cost that are related to the short-term planning policy are not considered in this model. Those issues can be discussed separately in a subsequent stage with various inventory models.

- The demands of multiple-products are uncertain but we assume the probability distribution is known. It is also assumed that the market demands for the products are independent of each other.
- For each product, the unit understocking and overstocking costs are available.
- Suppliers offer all-unit quantity discount prices, which depend on the purchased amount of the order on a single raw material from the supplier.

The following notations are used in our subsequent models:
Indices:
$k =$ 1,..., K: index of products
$i =$ 1,..., I: index of raw material
$j =$ 1,..., J: index of suppliers
$l =$ 1,..., L_j: index of price level offered by supplier j.

Parameters:
e_k = unit production cost for product k
r_k = unit sales revenue of product k
g_{ik} = the number of units of raw material i required to produce one unit of product k
t_k = the resource consumed by the manufacturer to produce one unit of product k
n_{ij} = the amount of supplier j's internal resource required to produce one unit raw material i
Q = the resource capacity of manufacturer
c_{ijl} = the unit prices of raw material i after discount from supplier j on discount segment l
d^S_{ijl} = the lower bound of the quantity of raw material i from supplier j on discount segment l
d^H_{ijl} = the upper bound of the quantity of raw material i from supplier j on discount segment l
z_k = the random variable of the demand for product k
$f(z_k)$ = the probability density function followed by the demand of product k
a_k = the estimated understocking cost of one unit of product k
b_k = the estimated overstocking cost of one unit of product k
m_j = the management cost associated with supplier j.

The management cost, m_j, is the cost to maintain a supplier j in the manufacturer's supplier base. The management cost refers to the cost created by any supplier management/development activity, such as supplier performance evaluation, supplier quality improvement, and includes any other efforts to keep the supplier qualified for the raw material supply.

The resource, n_{ij}, t_j, can be any productive resource or capacity usage as long as the units for these two variables match the ones for parameters q, Q respectively. It can actually refer to any physical resource that contributes to the production and is crucial for limiting suppliers or the manufacturer in their production capacities.

We define the following decision variables:

y_k : the amount of product k produced during the planning horizon

x_{ijl} : the amount of raw material i purchased from supplier j on quantity discount segment l

u_{ijl} : 1 if the manufacturer buys any raw material i from supplier j at price level l; otherwise 0

v_{ij} : 1 if manufacturer buys any raw material i from supplier j; otherwise 0

w_j : 1 if supplier j is chosen for any raw material purchase; otherwise 0.

Thus, the objective of maximizing the manufacturer's benefit is formulated as:

$$Max \sum_{k=1}^{K}\{\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k + \int_{y_k}^{\infty}[r_k y_k - a_k(z_k - y_k)]f(z_k)dz_k\} - \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L_j} c_{ijl}x_{ijl} - \sum_{k=1}^{K} e_k y_k - \sum_{j=1}^{J} m_j w_j. \tag{1}$$

The first item represents the manufacturer's expected revenue minus the overstock cost when the production amounts are above the actual demand levels. The second item is the manufacturer's expected revenue minus the shortage cost when the production amounts are lower than the actual market demands. The third item is the cost for purchasing the raw material. The fourth item is the production cost, and the last item is the management cost associated with suppliers.

The problem is subject to the following constraints:

- Raw material requirement constraints: to ensure there are enough raw materials for the production planning in the period

$$\sum_{k=1}^{K} m_{ik} y_k \leq \sum_{j=1}^{J}\sum_{l=1}^{L_j} x_{ijl}, \quad i = 1, ..., I. \tag{2}$$

- Capacity limits constraints:

$$\sum_{k=1}^{K} t_k y_k \leq Q, \tag{3}$$

$$\sum_{i=1}^{I} n_{ij} \sum_{l=1}^{L_j} x_{ijl} \leq q_j w_j, \quad j = 1, ..., J. \tag{4}$$

Constraint (3) is the manufacturer's crucial capacity restriction for the production planning. Constraints (4) ensure the resources of a supplier spent for producing the raw material should not exceed the total amount of resource reserved by the supplier for the manufacturer.

- Quantity discount constraints: The suppliers provide an all-unit quantity discount scheme. Here, we have the following constraints:

$$x_{ijl} \leq d^H_{ijl} u_{ijl}, \quad \forall i, j, l, \tag{5}$$

$$x_{ijl} \geq d^S_{ijl} u_{ijl}, \quad \forall i, j, l, \tag{6}$$

$$\sum_{l=1}^{L_j} u_{ijl} = v_{ij}, \quad \forall i, j. \tag{7}$$

(5) and (6) ensure the amount purchased from the supplier at the price level positions in the discount interval offered. Constraints (7) ensure only one discount level is eventually applied to the amount if the manufacturer purchases the raw material i from the supplier j.

- Management cost constraints: The supplier management cost will be generated if the supplier is selected to provide any raw material.

$$w_j \geq v_{ij}, \quad \forall i, j. \tag{8}$$

- Other nonnegative and integral constraints:

$$x_{ijl}, y_k \geq 0, \forall i, j, l, k; \tag{9}$$

$$u_{ijl}, v_j, w_j \in \{0, 1\}, \quad \forall i, j, l. \tag{10}$$

The formulation given by (1) to (10) forms a complete mixed integer nonlinear programming model for the problem. Since the supply of a raw material is allowed to be split among multiple suppliers in the model, we refer to the model as *Multiple-sourcing model.* Sometimes, the manufacturer may only choose one or a limited number of suppliers for each raw material. If so, two new models are introduced that are slightly modified from the aforementioned model.

Single-sourcing Model: If there is only single supplier selected for one raw material type, the following single-sourcing constraints are necessary for the model to meet this requirement.

$$\sum_{j=1}^{J} v_{ij} = 1, \quad \forall i. \tag{11}$$

Specific Multiple-sourcing Model: If at most N ($N \geq 2$) suppliers for the provision of one raw material is allowed, the following specific multiple-sourcing constraints must be added to the model given by (1) to (10).

$$\sum_{j=1}^{J} v_{ij} \leq N, \quad \forall i. \tag{12}$$

4 Solution approach

The aforementioned MINLP model is difficult to solve not only because a general MINLP problem is hard to solve but also the current MINLP solvers are unable to deal with the integration function, which is employed here to formulate expected revenue for uncertain demand. Such functions to calculate expected value usually cannot be expressed as a closed form. Here we investigate a solution approach, which extends the ability of the solvers for deterministic MINLP to solve the MINLP model with an expected objective function. The advantage of such an approach is that we can exploit the ability of current solvers and develop a problem independent approach, which can be employed to solve many similar MINLP problems – even stochastic 0-1 programming problems.

GAMS, with its integrated multiple solvers for MINLP, was ultimately chosen as a suitable package for tackling this problem. The solution approach employed consists of GAMS modelling language, SBB, nonlinear programming solvers, and external functions coded in C language. GAMS provides a modelling language and interface to different parts. SBB is one of the GAMS solvers for MINLP models. It is based on the combination of the standard branch-and-bound method and some NLP solvers, such as CONOPT, MINOS, and SNOPT. At each node of the search tree in the SBB branch-and-bound algorithm, the relaxed problem is an NLP model, which can be solved by one of the NLP solvers (According to our experiment, CONOPT demonstrated good performance for the examples).

[Insert figure 2 about here]

The structure of the GAMS solution program for solving the MINLP model is illustrated in Figure 2, which includes the GAMS main program and an external module coded with C language. The main program formulates the model discussed in the last section, except the integration part of the objective function. In the external module, an advanced C program routine was developed as an external function to define the following integration function:

$$\sum_{k=1}^{K}\{\int_{0}^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k + \int_{y_k}^{\infty}[r_k y_k - a_k(z_k - y_k)]f(z_k)dz_k\}. \tag{13}$$

This piece of C code performs numerical integration evaluation by embracing the Romberg integration algorithm - originally coded by Press *et al.* (1992), and modified slightly for employment in this study. The Romberg algorithm is a sophisticated numerical integration technique that uses the extended trapezoidal rule. The external module is then integrated into the GAMS main program by a GAMS interface. Moreover, the external module provides the first order derivative of the integration function, which is linked to the NLP solver through GAMS's external function interface.

Since the model discussed in Kim *et al.* (2002) is a special case of our problem, we employed this GAMS based solution method to solve the two examples given in their paper. Not surprisingly, the results are the same as those reported in the paper.

5 Computational testing and managerial analysis

This section presents main results of our numerical experiences. The computational experience is conducted on a PC with Windows XP (Pentium D CPU 280, 1GB of RAM).

The first two examples were originally introduced in Kim *et al.* (2002). Example 1 is a real-world case: a computer company assembles and sells two models. The manufacturer has 2 products and 5 components from 4 suppliers for production. In Example 2, there are 5 products, 5 components, and 5 suppliers. We design the quantity discount schemes and management costs, which are new parameters introduced in our models, for the two examples.

To verify the performance of the solution approach for large-scale problems, we construct Example 3 - a large scale problem with 10 products, 20 components, and 12 suppliers. The data for all examples are available from the authors on request.

5.1 Numerical results - the optimal acquisition policy

To demonstrate the optimal acquisition policy, a complete solution for each case is discussed follows. The running times to solve the problems are less than 1 minute for both numerical examples.

The solution for the single-sourcing model on Example 1 is illustrated in Table 1, where the manufacturer's capacity is set as $Q = 3200$ and the supplier 2's capacity $q_2 = 40$.

[Insert table 1 about here]

From the table, we can observe the following managerial implications:

- The optimal purchasing policy for the manufacturer from the above solution is, for example, purchasing 19.389 Component 1 from Supplier 1 at discount price, purchasing 20.011 Component 2 from Supplier 1 at discount price in the planning horizon, and so on. The manufacturer makes a contract with only one vendor for each component due to the single-sourcing restriction.

- The optimal production levels of 19.389 for Product 1 and 20.011 for Product 2 over the planning period are suggested. These levels are lower than the means of the product's demands.

- With the optimal purchasing policy and production level, the manufacturer would be able to earn a profit of \$4770.136.

Table 2 shows the complete solution for a multiple-sourcing model on Example 2, where the manufacturer's and the fourth supplier's capacities are set as $Q = 2000$ and $q_4 = 1500$ respectively (The default value for q_4 is 6000 and here it is changed to 1500 to observe a multiple-sourcing order scenario). It is noted that the manufacturer purchases Component 1 from both vendor 1 and vender 4. Even though the price of vender 4 is lower than that of vendor 1, the solution does not suggest using the full capacity of vender 4 since more purchasing from vender 1 at the discount price results in increased cost savings.

[Insert table 2 about here]

Table 3 summaries the solution and computation experience over the large scale problem with two different manufacturer's capacities, for both single- and multiple-sourcing policies. From the table, one can see that the running times are between 3027 to 6869 seconds, which are significantly longer that that for small problems, though the running time of a couple of hours is acceptable to determine long-term optimal acquisition policy.

[Insert table 3 about here]

5.2 Managerial analysis and comparisons

In this section, we investigate the relationship between solutions and some critical factors, using Example 2 as the illustrated example except as noted.

The manufacturer's profit versus capacity, and optimal capacity

We observed how the manufacturer's profit changes as its capacity (Q) varies from 1600 to 2400 ($q_4 = 6000$). The numerical results shows that the manufacturer's profit increases as the capacity increases but stays unchanged after about $Q = 2100$, even if the capacity increases further. This is because the manufacturer does not fully utilize the extra capacity due to the limitation of demands. We have the following proposition to present the property.

Proposition: If there is a solution to the given problem of (1) to (10), then there exists a capacity Q^* such as the profit increases as the capacity Q increases when $Q < Q^*$ and keeps unchanged after more than Q^*.

Proof. Let $P(y_k)$ denote the manufacturer's profit formulated as (1). Then we introduce the Lagrangian function by relaxing the constraints related to y_k and Q:

$$L = P(y_k) + \lambda(Q - \sum_{k=1}^{K} t_k y_k) + \sum_{i=1}^{I} \eta_i (\sum_{j=1}^{J} x_{ij} - \sum_{k=1}^{K} m_{ik} y_k), \quad (14)$$

where, $\lambda \geq 0$, and $\eta_i \geq 0, i = 1, ..., I$, are Lagrangian multipliers associated the related constraints.

The optimum solution $y_k, k = 1, ..., K$ should satisfy following Karush-Kuhn-Tucher conditions:

$$\partial L/\partial y_k = r_k + a_k - (r_k + a_k + b_k)F(y_k) - e_k - t_k\lambda - \sum_{i=1}^{I} \eta_i m_{ik} = 0, \quad (15)$$

$$for \;\; j = 1, ..., J$$

$$\lambda(Q - \sum_{k=1}^{K} t_k y_k) = 0, \quad (16)$$

$$\sum_{k=1}^{K} t_k y_k \le Q, \quad (17)$$

$$y_k \ge 0, \quad (18)$$

where $F(y_k)$ is a cumulative density function. We have $\partial L/\partial Q = \lambda$. As the meaning of Lagrangian multipliers discussed in the texts, it gives the profit increase with an unit of capacity increase. According to the complementarity condition (16), $\lambda \ge 0$ when the constraint is tight; otherwise $\lambda = 0$. It means the profit will not increase as Q increases ($Q > \sum_{k=1}^{K} t_k y_k$). Next we show $\lambda > 0$ when the constraint is tight except at Q^*.

From (15) we have

$$[r_k + a_k - (r_k + a_k + b_k)F(y_k) - e_k - t_k - \sum_{i=1}^{I} \eta_i m_{ik}]/t_k = \lambda. \quad (19)$$

Equation (19) states that the marginal utility per unit of allocation capacity for all products is λ. To let $\lambda = 0$, we must have

$$y_k^* = F^{-1}(\frac{r_k + a_k - e_k - t_k - \sum_{i=1}^{I} \eta_i m_{ik}}{r_k + a_k + b_k}). \quad (20)$$

It is straightforward to show that the objective is concave, and all constraints are linear. Then the solution that satisfied KKT conditions is the global optimum. Since $F(y_k)$ is a continuous increasing monotonic function, the solution y_k^* is unique. Let $Q^* = \sum_{k=1}^{K} t_k y_k^*$. Then we conclude that $\lambda = 0$ only if $Q \ge Q*$ and $\lambda > 0$ when $Q < Q*$. □

To detect the optimal capacity that allows the manufacturer to gain the maximal profit but without excess capacity, we revise the model by setting Q as decision variable and change the constraint (3) to $\sum_{k=0}^{K} h_k y_k = Q$, then solve the problem. This will provide optimal production levels, acquisition policy, and the optimal manufacturer's capacity simultaneously. For Example 2 with given parameters as the appendix, the problem is solved with the optimal capacity $Q^* = 2048.2$. The optimal capacity under uncertain demand provides the manufacturer crucial information for the strategic decision regarding the size of its capacity. With this information, the manufacturer could adjust the capacity during the planning period. It could, for example, request extra capacity if needed.

The acquisition pattern versus vendor's capacity

Since material costs are linear, the manufacturer selects the supplier who offers the lowest price for each component. But this is not always feasible due to multiple components and capacity limits. Table 3 compares the manufacturer's optimal order quantities for Component

1 as Supplier 4's capacity q_4 takes three different values: 1500, 2000, and 2500, given $Q = 2000$. For $q_4 = 1500$, the manufacturer orders Component 1 from vendors 1 and 4. Note that $c_{142} < c_{122} < c_{112}$ and the amount ordered from vendor 4 has not reached the capacity limit, even when it offers a lower price than vendors 1 and 2. However, the order from vendor 1 is equal to the discount break point. Thus the order from vendor 1 at the discount price results in higher cost savings. The solution does not recommend ordering Component 1 from vendor 2 simply because supplier 2 is chosen to supply Component 4 and does not have enough capacity for Component 1. For $q_4 = 2000$, the solution suggests to order the majority of Component 1 from vendor 4 at its capacity and a little from vendor 2. For $q_4 = 2500$, vendor 4 is chosen to supply whole component 1 since it has the lowest price and enough capacity.

[Insert table 5 about here]

The acquisition pattern also depends on the management cost. For example, our experiment shows that the manufacturer originally ordering 2444.337 units of Component 4 from supplier 2 at management costs $m(j) = 350, j = 1, ..., 5$, given $Q = 2000$ and $q_4 = 2500$, but then changing the order from supplier 5 when the management costs $m(j), j = 1, ..., 5$, increase to 1400. The saving of management cost of supplier 2 (now nothing is ordered from the vendor) will compensate the increased cost from supplier 5 due to a marginally higher price of component 4.

The production amount versus product demand

Figure 3 shows that the production amount y_3 increases linearly when its demand mean μ_3 increases, given $Q = 2000$. The production amount of other products remains constant when μ_3 increases from 130 to 150, and decreases μ_3 increases from 150 to 220. Calculating the optimal capacity provides us the insight to explain the results. The optimal capacity Q^* is 1998.004 for $\mu_3 = 150$ and 2017.883 for $\mu_3 = 160$ respectively. This implies that the manufacturer has redundant capacity at optimal production levels when $\mu_3 \leq 150$. At this point, the increasing production of product 3 does not affect production of other products, while it does when $\mu_3 \geq 160$ since the capacity for optimal production levels is higher than the manufacturer's capacity. The manufacturer's profit increases linearly when its demand mean μ_3 increases, given $Q = 2000$.

[Insert figure 3 about here]

Comparison of the results of discount case with that of non-discount case

We compare our acquisition policy with that of Kim *et al.* (2002) for the case of multiple-sourcing policy, which is only policy employed in their model. It is noted that the profit is not comparable since our model takes into account the management cost and price discounts from suppliers. Table 4 shows comparisons of acquisition policy for Q =1500, 2000, and 2500. For all those solutions, supplier 1 is selected for components 1 and 2; supplier 2 for 3; supplier 4 for 4 and 5. But the order quantities from each supplier are different. When $Q = 1500$, the solutions suggest to shift capacity from product 1 to product 2 in discount case instead of producing small amount of product 1 in non-discount case. It is partly because product 2 has higher profit under discount case. When $Q = 2000$, comparing with non-discount case, the production level of product 1 increases from 7.637 to 10.001, which is the threshold for discount. For $Q = 2500$, the allocation of acquisition policy is similar for both cases. In summary, the discount schemes do have impact to acquisition policy and production level.

[Insert table 4 about here]

Comparisons of single- and multiple-sourcing policies

Figure 4 compares the optimal procurement of single- and multiple-sourcing policies when $Q = 1500$ and $q_4 = 1500$. It is shown that with the single-sourcing policy, the manufacturer orders Component 1 only from supplier 1, while the solution of the multiple-sourcing policy suggests the manufacturer to acquire Component 1 from both supplier 1 and Supplier 4. The amount of 1000.001 from supplier 1 enables the buyer to enjoy the benefit of discount price. Consequently, the profit of the solution from multiple-sourcing model, valued as 4700, is higher than that of the solution from single-sourcing model, valued at 4600.

[Insert figure 4 about here]

6 An extension to volume discount schemes

There are different discount schemes suppliers can consider and these choices relate highly to how the supplier want to promote their products. Different from quantity discount, volume discount is based the total value amount of different products purchased, which is utilized when the supplier wants to encourage the manufacturer to order multiple raw material types instead of particular products. We have presented a MINLP model to optimize acquisition policy and production levels under uncertain demand and volume discounts. The objective of maximizing the manufacturer's benefit is formulated as:

$$Max \ \sum_{k=1}^{K}\{\int_{0}^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k + \int_{y_k}^{\infty}[r_k y_k - a_k(z_k - y_k)]f(z_k)dz_k\}$$
$$-\sum_{j=1}^{J}(\sum_{i=1}^{I} c_{ij}x_{ij})\sum_{l=1}^{L_j}(1-d_{jl})u_{jl} - \sum_{k=1}^{K} e_k y_k - \sum_{j=1}^{J} m_j w_j. \quad (21)$$

This is the same as that for quantity discount models except the purchasing cost item. We introduce d_{jl} to represent the volume discount rate on segment l from supplier j. $c_{ij}x_{ij}$ implies volume value. The constraints for the problem are:

$$\sum_{k=1}^{K} m_{ik}y_k \leq \sum_{j=1}^{J} x_{ij}, \ \forall i, \quad (22)$$
$$\sum_{k=1}^{K} t_k y_k \leq Q, \quad (23)$$

$$\sum_{i=1}^{I} n_{ij} x_{ij} \leq q_j w_j, \quad \forall j, \tag{24}$$

$$\sum_{i=1}^{I} c_{ij} x_{ij} \leq \sum_{l=1}^{L_j} V_{jl}^{H} u_{jl}, \quad \forall j, \tag{25}$$

$$\sum_{i=1}^{I} c_{ij} x_{ij} \geq \sum_{l=1}^{L_j} V_{jl}^{S} u_{jl}, \quad \forall j, \tag{26}$$

$$\sum_{l=1}^{L_j} u_{jl} = w_j, \quad \forall j, \tag{27}$$

$$w_j \geq v_{ij}, \quad \forall i, j, \tag{28}$$

$$x_{ij} \leq M v_{ij}, \quad \forall i, j, \tag{29}$$

$$x_{ij}, y_k \geq 0, \forall i, j, k, \tag{30}$$

$$u_{jl}, v_j, w_j \in \{0, 1\}, \quad \forall j, l. \tag{31}$$

The notations used in this model are either the same or straightforward change from those for the quantity model. For example, we use u_{jl} for the decision variable that is 1 if supplier j offers volume discount at segment l, V_{jl}^{H} and V_{jl}^{H} for the upper and lower bounds of the segment l.

A similar solution approach is implemented to solve the acquisition policy decision problem with volume discounts formulated as (21) to (31). We have employed the model and solution approach to an volume discount example. Numerical results indicate both the model and the solution method are effective for the problem.

7 Conclusions and future work

This study presents a mixed integer nonlinear programming (MINLP) approach that simultaneously determines the production level of multiple-products, supplier selection, and order quantity allocation for a manufacturer under uncertain demands, where suppliers offer quantity discount schemes. MINLP models are developed to maximize the expected value of the manufacturer's profit with both single- and multiple-sourcing policies, subject to both the manufacturer's and suppliers' capacities. The GAMS-based solution program is developed and verified to be capable of efficiently solving the MINLP models. The primary numerical results and managerial analysis for two examples are reported.

The several variants of the model have been developed and utilized to find optimal acquisition policy for different scenarios, for example, dealing with single- or multiple-sourcing policies. A variant has also employed to find a manufacturer's optimal capacity under uncertain demands, one of manufacturer's strategic decision problems. An extension to volume discounts is discussed in this paper.

Another extension to the MINLP model could combine the strategic decision of the vendor selection problem with short-term inventory management to increase a manufacturer's total benefit and the supply network's efficiency under an uncertain demand environment.

Although the solution method developed in this paper works well for the two small and one large scale examples, more investigation on effective algorithms would be valuable for solving such MINLP problems involving large number of products and suppliers. Besides, the study to extend the approach to other stochastic 0-1 programming would undoubtedly interest to both practitioners and academic researchers.

Acknowledgments

This research is supported by NSERC Discovery grant and CFI funding. The second author thanks the Government of Ontario for an OGSST Scholarship. The authors would like to thank the two referees for their helpful comments and suggestions.

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Table 1: The complete solution to Example 1 with $Q = 3200$ and $q_2 = 40$

	Production levels		Purchase Amounts				
			Comp 1	Comp 2	Comp 3	Comp 4	Comp 5
Profit	y_1	y_2	Supp 1	Supp 1	Supp 2	Supp 4	Supp 4
4770.136	19.389	20.011	19.389	20.011	39.400	19.389	20.011

Table 2: The complete solution to Example 2 with $Q = 2000$ and $q_4 = 1500$

Profit	Production levels				
	y_1	y_2	y_3	y_4	y_5
67122.629	217.684	177.872	201.124	189.251	215.274

	Purchase quantity					
Component i	1		2	3	4	5
Supplier j	1	4	5	1	2	3
Amount	1000.001	1051.685	1773.347	2182.197	2444.337	2177.872

Table 3: The solution summary to Example 3

	Q=4000			Q=5500		
Policy	Profit	Nodes	Seconds	Profit	Nodes	Seconds
single-sourcing	251920.03	531	6869	268156.08	391	5499
multiple-sourcing	264184.69	301	3027	280375.67	462	5734

Table 4: Comparisons of optimal order quantities with that of Kim *et al.* 2002

Q	model	Profit	y_1	y_2	comp1	comp2	comp3	comp4	comp5
1500	Kim	1856.657	1.387	17.363	1.387	17.363	18.750	1.387	17.363
	Discount	2554.091	0	18.750	0	18.750	18.750	0	18.750
2000	Kim	2569.161	7.637	17.363	7.637	17.363	25.000	7.637	17.363
	Discount	3453.414	10.001	14.999	10.001	14.999	25.000	10.001	14.999
2500	Kim	3279.996	13.767	17.483	13.767	17.483	31.250	13.767	17.483
	Discount	4479.732	12.833	18.417	12.833	18.417	31.250	12.833	18.417

Table 5: A comparison of optimal order quantities for Component 1, when $Q = 2000$

Capacity q_4	Supplier j	Amount
1500	1	1000.001
	4	1051.685
2000	2	44.827
	4	2000.000
2500	4	2051.686

CAPTIONS

Figure 1. A supply network involving one manufacturer and multiple suppliers

Figure 2. The structure of the solution program

Figure 3. The production amount versus product demand

Figure 4. A Comparison of single- and multiple-sourcing procurement policies

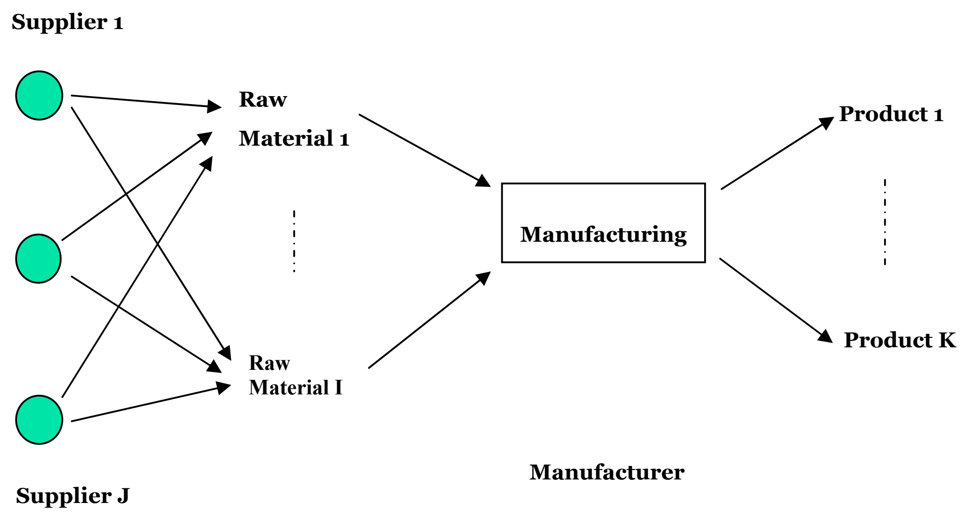

Figure 1: A supply network involving one manufacturer and multiple suppliers

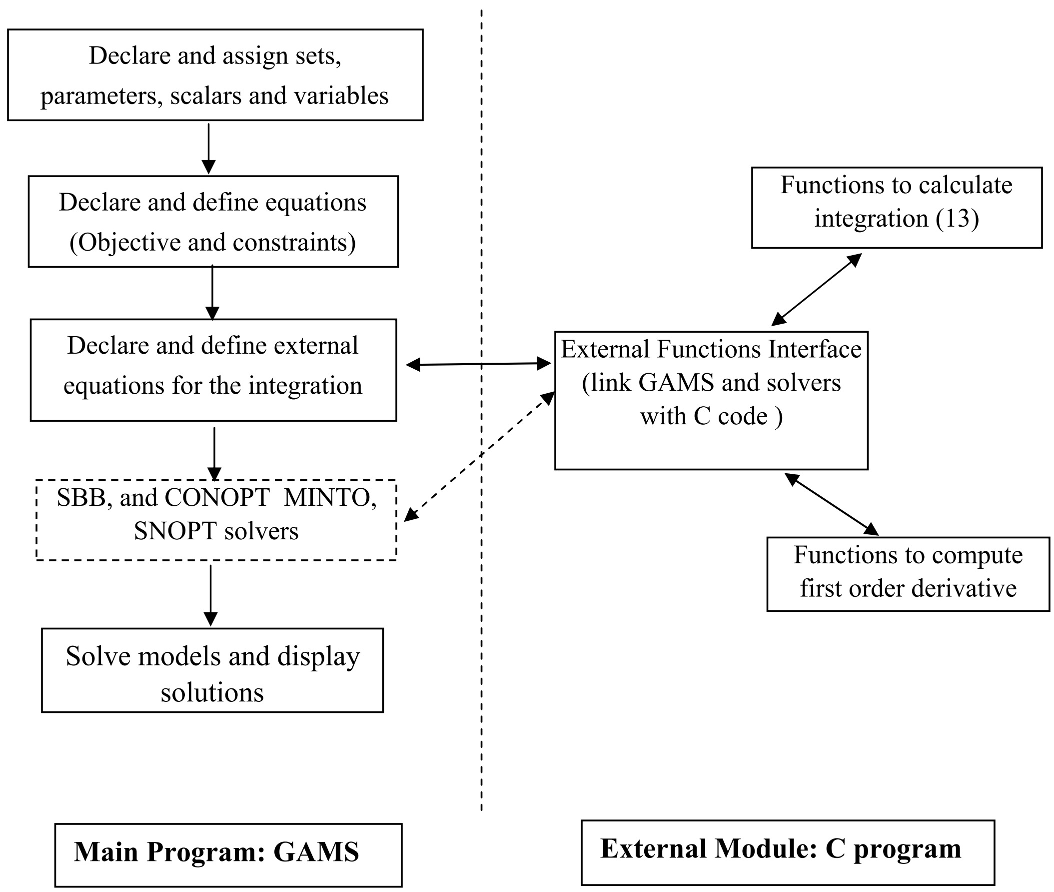

Figure 2: The structure of the solution program

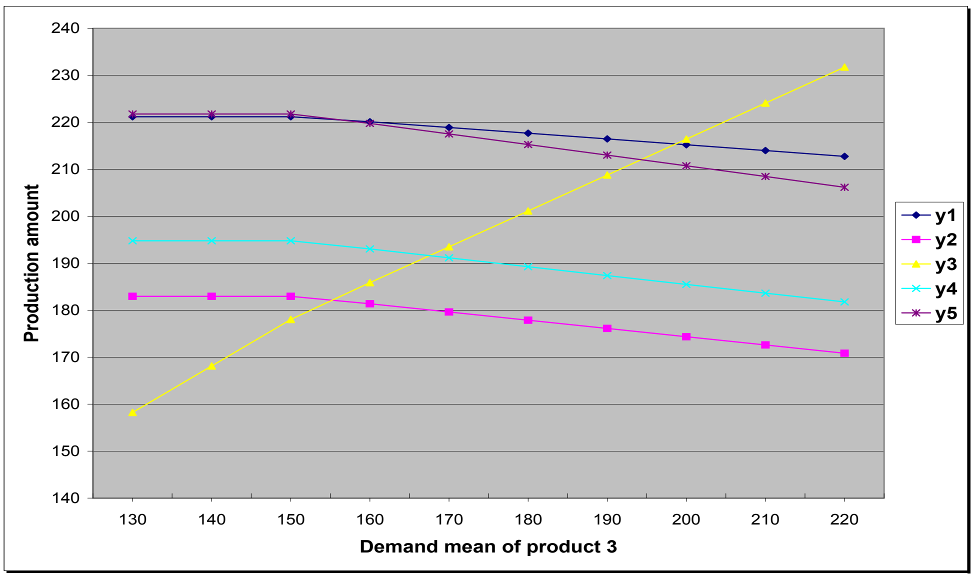

Figure 3: The production amount versus product demand

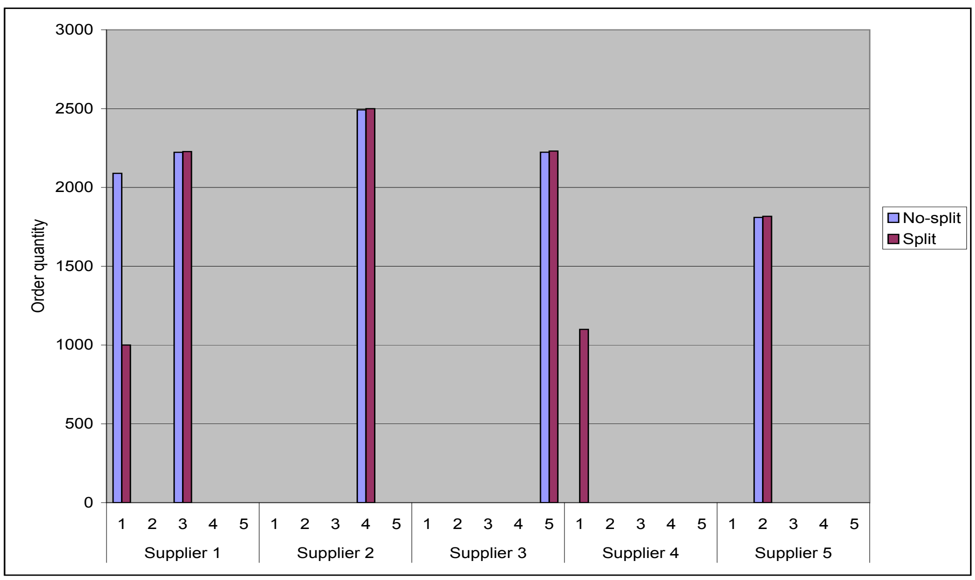

Figure 4: A comparison of single- and multiple-sourcing policies