

Tools for Voltage Collapse Assessment

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Abstract—A variety of tools and techniques used for voltage stability analysis in power systems are discussed in this paper, with particular emphasis on their application to electricity markets. Thus, the computation of transfer capability limits, which are directly associated with the security constraints used in energy auction systems, by means of continuation power flows (CPF) as well as maximum loadability optimal power flows (OPF) are discussed in some detail. The paper also describes a “standard” security-constrained (SC) OPF auction, as well as two recently proposed voltage-stability-constrained (VSC) OPF-based techniques used to better represent security limits in market clearing and power dispatch techniques. A simple 6 bus example is utilized to illustrate the application of all these methodologies, and discuss and compare the various market prices, power and system security levels obtained from the different techniques, showing the advantages of the VSC-OPF techniques over the standard SC-OPF.

Index Terms—Voltage stability, maximum loadability, transfer capability, continuation power flow, OPF, congestion management, electricity markets.

I. INTRODUCTION

Voltage stability has been recognized as one of the main causes for major blackouts throughout the world in the recent past [1]. For example, one of the main technical causes of the August 2003 blackout of the Canada-US power grid in North America’s North East was lack of sufficient reactive power support, i.e. a voltage stability problem [2]. The continuous increase in power demand, the limited expansion of transmission systems due to environmental and economic reasons, and the relatively new problem of unpredictable generation dispatch patterns and insufficient reactive power support directly associated with the processes of deregulation/privatization of electricity markets, all have led to a reduction in voltage stability margins of power grids. As a matter of fact, voltage stability has become the main technical factor limiting the power transfer capacity of transmission systems in many electricity markets (e.g. Ontario). Hence, the proper accounting and management of voltage stability has become of great importance for the adequate and reliable operation of electricity markets and their associated power grids.

Voltage stability is a well known and studied phenomenon [3]. A series of mature and well-developed tools, such as continuation power flows, are widely used by system operators to determine the transfer capability of power networks. These transfer capabilities are computed off-line assuming a variety

of generation and load dispatch patterns, and these limits are then used in clearing and dispatch algorithms to select the generators for supplying the demand, which for the most part is inelastic, and to determine electricity prices. Since dispatch patterns and prices are directly affected by these limits, and these limits are at the same time affected by the dispatch patterns, the current off-line procedures used to determine transfer limits are simply inadequate, as discussed in detail in [4]. More recently, in [5], [6], two novel techniques to directly represent voltage stability in optimal power flow (OPF)-based dispatch methodologies were proposed; these techniques are referred here to as Voltage Stability Constrained (VSC)-OPF. The multi-objective VSC-OPF methodology proposed in [6] is used in [4] to study the effect of the proper representation of voltage stability margins on market electricity prices, dispatch patterns and security levels; this technique is presented and discussed in the current paper for comparison purposes. The VSC-OPF technique proposed in [5], which is based on a singular-value voltage stability index, is used in this paper as an OPF-based clearing and dispatch methodology to compare it against the technique discussed in [4].

This paper is structured as follows: Section II discusses the basic concepts and tools behind the computation of transfer capacity limits in electricity markets. In Section III, the “classical” OPF-based auction model used for clearing and dispatch in electricity markets is described in detail, together with the two VSC-OPF models presented here to better represent voltage stability margins in an OPF-based auction model. Section IV discusses and compares the results of applying the three OPF-based action models described in Section III to a 6-bus test system model with supply and demand side bidding. Finally, Section V summarizes the main results and contributions of this paper.

II. TRANSFER CAPABILITY COMPUTATIONS

System security requirements basically impose limitations on the power transfer capabilities of the transmission system. In electricity markets, the ISO needs to guarantee the feasibility of the potential transactions, and only those transactions that are within the transfer capabilities of the network are allowed, thus dispatching units without violating transmission security limits. Typically, the transfer capability of the main transmission system corridors are defined based on the concept of Available Transfer Capability (*ATC*), which is in turn defined as the remaining transfer capability of the transmission

system for further commercial activity over and above already committed uses [7], and is commonly expressed as:

$$ATC = TTC - TRM - CBM - ETC \quad (1)$$

where TTC is the Total Transfer Capability; TRM is the Transmission Reliability Margin, which is typically assumed to be a fixed value (e.g. 5% of TTC under normal operating conditions in the Western Electricity Coordinating Council WECC [8]); CBM is the Capacity Benefit Margin and is assumed here to be equal to zero; and ETC is the Existing Transmission Commitments. The TTC is typically defined as follows:

$$TTC = \max \left\{ P_{\max_{I_{lim}}}, P_{\max_{V_{lim}}}, P_{\max_{S_{lim}}} \right\} \quad (2)$$

where $P_{\max_{I_{lim}}}$, $P_{\max_{V_{lim}}}$ and $P_{\max_{S_{lim}}}$ stand for the maximum powers the system can securely transmit considering thermal limits, voltage limits, and stability limits, respectively.

Currently, electricity markets are being operated under stressed loading conditions due to the increased demand and power transfer levels, thereby increasing the probability for the system to experience stability problems. Under such conditions, system stability limits can be approximated through voltage stability limits, which correspond to loading points at which power flow solutions “disappear” due to the Jacobian of the power flow equations becoming singular or certain voltage controls reaching their limits (e.g. generators’ reactive power limits [3]). In this context, the TTC basically represents the maximum power in a given transmission corridor, for a given dispatch pattern, which does not violate thermal limits, bus voltages limits or voltage stability limits [9]. Hence, assuming a $TRM = 0.05 * TTC$, the ATC can thus be approximated as follows:

$$ATC = 0.95 TTC - ETC \quad (3)$$

The ATC values are typically calculated for the “worst” contingency, based on N-1 or N-2 contingency criteria, by means of the system PV curves, which allow to compute the change in power levels between the operating point and the “tip” of the curves, i.e. the system maximum loading levels associated with thermal, voltage magnitude or voltage stability limits. These PV curves can be obtained using continuation power flow (CPF) methods [3], which allow calculating complete voltage profiles for a given dispatch, i.e. a “direction” of generation and loading change, and considering different contingencies. On the other hand, the maximum loading can also be computed by reformulating the conventional optimal power flow (OPF) as a load maximization problem [3], [6]. CPF and OPF techniques not only yield ATC values, but also provide sensitivity information through tangent vector and Lagrange multiplier values, respectively, which are a byproduct of these methods, and can be used to analyze the effect of changes in system parameters, such as loading, in the ATC values.

III. MARKET CLEARING MECHANISMS

A. Security-constrained OPF

A “standard” OPF-based auction model is basically defined as the following nonlinear programming problem, which consists of maximizing the linear objective function of social

benefit S_b , subject to a set of nonlinear equality and inequality constraints [4]:

$$\begin{aligned} \text{Min. } & S_b = -(C_d^T P_d - C_s^T P_s) \quad (4) \\ \text{s.t. } & F_{PF}(\delta, V, Q_G, P_s, P_d) = 0 \\ & 0 \leq P_s \leq P_{s\max} \\ & 0 \leq P_d \leq P_{d\max} \\ & |P_{ij}(\delta, V)| \leq P_{ij\max} \\ & I_{ij}(\delta, V) \leq I_{ij\max} \\ & Q_{G\min} \leq Q_G \leq Q_{G\max} \\ & V_{\min} \leq V \leq V_{\max} \end{aligned}$$

where C_s and C_d are vectors of supply and demand bids in \$/MW, respectively; P_s and P_d represent the supply and demand power levels in MW, respectively, with maximum bid limits; F_{PF} stands for the power flow equations of the system; V and δ define the bus phasor voltages; Q_G stands for the generator reactive powers; P_{ij} stands for the power flowing through the lines from bus i to j ($i \neq j$), which are used to represent system security by imposing limits on them, in addition to line current I_{ij} thermal limits and bus voltage limits.

In this model, which is referred to as a security-constrained (SC) OPF-based auction, P_{ij} limits are usually obtained by means of off-line loadability studies, as explained in Section II. Thus, Available Transfer Capability studies are performed for a variety of dispatch and load scenarios, considering the “worst” system contingencies, to determine these limits. Notice that these power transfer limits do not represent the actual stability conditions of the solutions obtained from the auction, since these limits are determined using operating conditions that do not necessarily correspond to actual solutions of the SC-OPF problem (4); this is particularly true if the bidding is such that the auction yields dispatch conditions not considered during the off-line ATC studies. This was not a significant issue before deregulation/privatization, as the cost curves of the different generators were known by system operators ahead of time, and hence dispatch schedules could be simulated and considered for a variety of typical loading conditions in off-line system security studies. Hence, this model may lead in some cases to insecure operating conditions and/or inappropriate price signals, as clearly illustrated in the simple example discussed in Section III.

B. Voltage-stability-constrained OPFs

To improve the representation of system security constraints in the standard OPF, various techniques to better represent voltage stability limits in the OPF are proposed and discussed in [4], [5].

In [5], the minimum singular value of the power flow Jacobian, which may be used as an index to predict voltage instability in power systems as discussed in [3], is introduced as an additional constraint in the OPF to properly account for voltage stability limits in the dispatch problem. Thus, a voltage-stability-constrained (VSC) OPF based on this index

can be written as follows [5]:

$$\begin{aligned}
\text{Min. } & S_b = -(C_d^T P_d - C_s^T P_s) & (5) \\
\text{s.t. } & F_{PF}(\delta, V, Q_G, P_s, P_d) = 0 \\
& 0 \leq P_s \leq P_{s_{\max}} \\
& 0 \leq P_d \leq P_{d_{\max}} \\
& \sigma_{\min}(J_{PF}) \geq \sigma_c \\
& I_{ij}(\delta, V) \leq I_{ij_{\max}} \\
& Q_{G_{\min}} \leq Q_G \leq Q_{G_{\max}} \\
& V_{\min} \leq V \leq V_{\max}
\end{aligned}$$

where $J_{PF} = [\partial F_{PF}/\partial \delta|_o \quad \partial F_{PF}/\partial V|_o]$ is the power flow Jacobian of the system at a power flow solution point $(\delta_o, V_o, Q_{G_o}, P_{s_o}, P_{d_o})$, and σ_c is a minimum limit for the σ_{\min} voltage stability index, so that voltage stability problems can be avoided even in the case of the worst contingencies; such a value would have to be determined by means of off-line studies, as discussed in [5].

The maximum system loadability level of the system is introduced in [4] as the security constraint in the OPF auction model, since this is a better measure of proximity to a voltage stability problem than the σ_{\min} index used in (5) [3]. This is accomplished through the following multi-objective optimization problem:

$$\begin{aligned}
\text{Min. } & G = -(1-w) \underbrace{(C_d^T P_d - C_s^T P_s)}_{S_b} - w \lambda_c & (6) \\
\text{s.t. } & F_{PF}(\delta, V, Q_G, P_s, P_d) = 0 \\
& F_{PF_c}(\delta_c, V_c, Q_{G_c}, P_s, P_d, \lambda_c) = 0 \\
& 0 \leq P_s \leq P_{s_{\max}} \\
& 0 \leq P_d \leq P_{d_{\max}} \\
& \lambda_{c_{\min}} \leq \lambda_c \leq \lambda_{c_{\max}} \\
& I_{ij}(\delta, V) \leq I_{ij_{\max}} \\
& Q_{G_{\min}} \leq Q_G \leq Q_{G_{\max}} \\
& V_{\min} \leq V \leq V_{\max} \\
& I_{ij_c}(\delta_c, V_c) \leq I_{ij_{c_{\max}}} \\
& Q_{G_{c_{\min}}} \leq Q_{G_c} \leq Q_{G_{c_{\max}}} \\
& V_{c_{\min}} \leq V_c \leq V_{c_{\max}}
\end{aligned}$$

In this case, a second set of power flow equations and constraints with a subscript c is introduced to represent the system at the limit or ‘‘critical’’ conditions associated with the maximum loading margin λ_c in p.u., where λ is the parameter that drives the system to its maximum loading condition, and is defined as follows:

$$\begin{aligned}
P_G &= P_{G_0} + P_s & (7) \\
P_L &= P_{L_0} + P_d \\
P_{G_c} &= (1 + \lambda_c + k_{G_c}) P_G \\
P_{L_c} &= (1 + \lambda_c) P_L
\end{aligned}$$

where P_{G_0} and P_{L_0} stand for generator and load powers which are not part of the market bidding (e.g. must-run generators, inelastic loads), and k_{G_c} represents a scalar variable used to distribute the system losses associated with the solution of

the critical power flow equations in proportion to the power injections obtained in the solution process, i.e. a standard distributed slack bus model is used. Observe that λ_c represents the maximum loadability of the network; hence, this value can be viewed as a measure of the congestion of the network and basically corresponds to the system’s *ATC*.

The limits on the loading margin λ_c are introduced in order to ensure a minimum and maximum levels of security in the auction model. These conditions ensure that the loading parameter remains within certain limits to avoid ‘‘unacceptable’’ solutions characterized by either low security levels ($\lambda_c < \lambda_{c_{\min}}$) or low supply and demand levels ($\lambda_c > \lambda_{c_{\max}}$). Furthermore, since in this model contingencies are not directly represented, the value of $\lambda_{c_{\min}}$ can be chosen so that contingencies can be indirectly considered through this value.

In the multi-objective function G , two terms are present, with their influence on the final solution being determined by the value of the weighting factor w ($1 > w > 0$). The first term represents the social benefit S_b , whereas the second term guarantees that the ‘‘distance’’ between the market solution and the critical point is maximized [6]. Observe that $w < 1$ so that the market bidding is properly represented in this OPF formulation, and that $w > 0$ so that λ_c does correspond to a maximum loading condition of the system. Notice also that the two terms of the objective function are expressed in different units, since S_b would be typically in \$/h, whereas the ‘‘security’’ term would be in p.u.; this affects the chosen value of w , which would typically be $w \ll 1$.

The VSC-OPF problems are computationally demanding when compared to the SC-OPF problem [4], [10], since (5) is a nonlinear programming (NLP) optimization problem that is twice as big as (4), while (6) is an NLP problem with an implicit constraint that requires special solution techniques, as explained in detail in [10].

IV. EXAMPLE

Figure 1 depicts the 6-bus test system used here. It is extracted from [4], where all system data can be found, and comprises three generation companies (GENCOs) and three energy supply companies (ESCOs) that provide supply and demand bids, respectively.

The results of applying the OPF formulation (4) to the test system are illustrated in Table I; the maximum loading or *TTC* value in this table was computed off-line using the generator voltages and load and generation power directions obtained from the OPF solution, with the help of a CPF, enforcing all thermal, voltage and reactive power limits and neglecting contingencies. On the other hand, Table II shows the solution obtained for the VSC-OPF (5), and Table III illustrates the solution obtained for the VSC-OPF (6). The values of σ_c and $\lambda_{c_{\min}}$ were chosen from the off-line maximum loadability studies used to determine the transmission line power limits used in (4), which correspond to a 95% maximum loading condition; thus, $\sigma_c = 0.25715$ and $\lambda_{c_{\min}} = 0.05$ p.u. ($\lambda_{c_{\max}} = 0.8$ p.u.) For all solutions, generator voltages are at their maximum limits, as expected, since this condition generally provides higher transactions levels. However, in

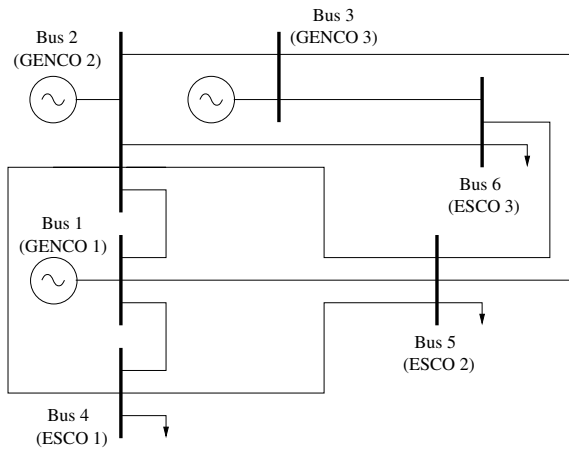


Fig. 1. 6-bus test system.

TABLE I
SC-OPF RESULTS

Participant	V [p.u.]	ρ [\$/MWh]	P_{BID} [MW]	P_0 [MW]	Pay [\$/h]
GENCO 1	1.100	9.70	14.4	90	-1013
GENCO 2	1.100	8.80	2.4	140	-1253
GENCO 3	1.084	8.28	20.0	60	-663
ESCO 1	1.028	11.64	15.6	90	1229
ESCO 2	1.013	10.83	0.0	100	1083
ESCO 3	1.023	9.13	20.0	90	1005
TOTALS	$T = 315.6$ MW Losses = 11.2 MW		$\text{Pay}_{\text{ISO}} = 388$ \$/h $TTC = 520$ MW		

comparison with the SC-OPF approach, the solutions of both VSC-OPF models, which are practically the same, provide better LMPs ρ , higher total transaction levels T ($T = \sum_i P_{L_i}$) and higher TTC values, which clearly demonstrates that off-line power flow limits are not adequate for representing the actual system congestion. The improved LMPs result also in a lower total price paid to the ISO (Pay_{ISO}), i.e. the network congestion costs are lower, even though the system losses are higher, which is to be expected, since the T values are higher.

V. CONCLUSIONS

The paper discusses in detail the influence of voltage stability in the operation of electricity markets and its associated grid. Recently developed tools that properly account for voltage stability in typical OPF-based clearing and dispatch mech-

TABLE II
VSC-OPF (5) RESULTS

Participant	V [p.u.]	ρ [\$/MWh]	P_{BID} [MW]	P_0 [MW]	Pay [\$/h]
GENCO 1	1.100	8.95	0.0	90	-805
GENCO 2	1.100	8.91	25.0	140	-1469
GENCO 3	1.100	9.07	20.0	60	-726
ESCO 1	1.021	9.49	25.0	90	1091
ESCO 2	1.013	9.58	10.0	100	1053
ESCO 3	1.039	9.35	8.0	90	917
TOTALS	$T = 323$ MW Losses = 11.9 MW		$\text{Pay}_{\text{ISO}} = 61.2$ \$/h $TTC = 539$ MW		

TABLE III
VSC-OPF (6) RESULTS FOR $w = 0.001$

Participant	V [p.u.]	ρ [\$/MWh]	P_{BID} [MW]	P_0 [MW]	Pay [\$/h]
GENCO 1	1.100	8.94	0.0	90	-805
GENCO 2	1.100	8.91	25.0	140	-1470
GENCO 3	1.100	9.07	20.0	60	-726
ESCO 1	1.021	9.49	25.0	90	1091
ESCO 2	1.013	9.57	10.0	100	1053
ESCO 3	1.039	9.35	8.0	90	916
TOTALS	$T = 323$ MW Losses = 12.0 MW		$\text{Pay}_{\text{ISO}} = 59$ \$/h $TTC = 539$ MW		

anisms used in electricity markets are also described. A simple example is used to illustrate the advantages of these new techniques with respect to a clearing and dispatch mechanism currently in used by some system operators, demonstrating that a better representation of system congestions through voltage stability indices that properly represent grid security levels, as opposed to using simple transfer limits computed off-line, lead to better market and system operating conditions.

The discussed VSC-OPF methods present some significant computational challenges that need to be resolved before these techniques can be applied in practice. Research is currently being conducted to develop computationally efficient techniques that would allow the use of the presented tools in real operating environments.

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