

Confidence Intervals Estimation in the Identification of Electromechanical Modes from Ambient Noise

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Abstract—This paper discusses the estimation of uncertainty intervals associated with the electromechanical modes identified from ambient data resulting from random load switching throughout the day in power systems. A connection between the second order statistical properties, including confidence intervals, of the identified electromechanical modes and the variance of the parameters of a selected linear model is demonstrated. The results of the presented method are compared with respect to the ones obtained from a Monte-Carlo type of simulation, showing its effectiveness in reducing the number of trials, which would be beneficial for on-line power system monitoring, as it can decrease the number of samples, thus ensuring that the system dynamics would not change significantly over the monitoring time window, and yielding more dependable results. Two test cases, i.e. the 2-area benchmark system and the IEEE 14-bus system, with different orders of the system identification model used (n), are utilized to demonstrate the effectiveness of the proposed methodology.

Index Terms—Power system oscillations, modal analysis, power system monitoring, system identification, prediction error methods.

I. INTRODUCTION

ON-LINE power system monitoring based on system identification techniques is of great help for providing insightful information regarding the stability condition of the system under study, as well as for validating off-line models and data. These techniques have been used to identify poorly damped electromechanical modes of a power system using simulations or field measurements, and are particularly relevant nowadays that phasor-measurement units (PMU) are being widely deployed and utilized.

Identification techniques and models are either based on the deterministic transient response of the system to a large disturbance or random ambient noise. The transient response of a power system is normally accompanied with ringdown tests and major disturbances, such as adding/removing loads, severe faults and tripping generators. For instance, the well-known Prony method, which employs a deterministic model, has been widely used in power systems to analyze this kind of response [1], [2], [3], [4]. On the other hand, ambient noise, which is a low quality signal, is the natural response of a power system due to small-magnitude, random load switching; thus, stochastic models such as auto-regressive (AR) models

[5], auto-regressive moving-average (ARMA) models [6], and stochastic state-space models [7] have been used in this case.

Identified electromechanical modes from stochastic models are usually represented by a mean value and the corresponding confidence interval, which are estimated by means of Monte-Carlo type of simulations or experiments [6], [7]. However, the drawback in this approach is that it requires repeating the experiments, which in turn can violate the stationarity assumption of the measured signal over a long time window, since system dynamics may undergo significant changes due to, for example, rescheduling, i.e. adding/removing new generator units. Therefore, experiments should be carried out in a time window as short as possible, which is what motivates the work presented in the current paper. The authors in [8] introduce a bootstrap method to give confidence interval estimates for electromechanical modes, and its performance is studied by comparing the results with respect to the ones obtained by means of Monte-Carlo type of simulations. This method requires resampling the measured data to estimate the parameters of the system model (e.g. ARMA) for the new data set, and is hence computationally expensive, since the resampling process is repeated for large number of trials in order to obtain proper estimates of the confidence intervals.

The electromechanical modes are the roots of the characteristic equation corresponding to the selected model (e.g. AR or ARMA); hence, there is a nonlinear relationship between the model parameters and modes, and between their corresponding variances. Therefore, the theory of the variance of parameters, which is well-understood and developed [9], may be used in this case to determine the variance of these modes. Reference [10] describes a technique that has been used in civil engineering for identification of structures to establish a connection between the variance of parameters and the variance of modal parameters. The application of this particular technique to estimate confidence intervals of electromechanical modes identified from ambient data is discussed here, showing that only one set of data may be used to estimate the mode uncertainties, thus reducing the required number of samples.

The rest of the paper is structured as follows: Section II presents the background on estimating the covariance of parameters of a linear parametric model within the context of prediction error methods (PEM) [11]; this information is then employed to estimate the confidence interval of the identified electromechanical modes. The results of applying the proposed technique for the 2-area benchmark system and the IEEE 14-bus system are presented in Section III. Finally, Section IV summarizes the main contributions of this paper.

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II. COVARIANCES

A linear parametric model such as AR or ARMA used for representing a power system presents the following typical form:

$$y(k) = H(q) e(k) \quad (1)$$

where $y(k)$ is the measured output, such as power through a line; $e(k)$ models the disturbances, i.e. the underlying load switching in a power system; $H(q) = B(q)/A(q)$ is a rational transfer function with unknown parameters θ ; and q is the shifting operator defined by $q^{-1}y[k] = y[k-1]$. One-step-ahead prediction $\hat{y}(k|\theta)$ uses the observations available up to time $k-1$ to predict $y(k)$, thus:

$$\hat{y}(k|\theta) = [1 - H^{-1}(q)] y(k) \quad (2)$$

This notation is adopted from [9], and is used here to emphasize the dependence on the parameter vector θ . For instance, for an ARMA(p,d) model described as:

$$y(k) = -\sum_{i=1}^p a_i y(k-i) + \sum_{i=0}^d b_i e(k-i) \quad (3)$$

vector $\theta = [a_1 \dots a_p \ b_0 \ b_1 \dots b_d]^T$ may be computed, within the context of PEMs, by minimizing an objective function such as:

$$\begin{aligned} \hat{\theta} &= \arg \min V_N(\theta) \\ V_N(\theta) &= \frac{1}{N} \sum_{k=1}^N \frac{1}{2} \epsilon^2(k, \theta) \\ \epsilon(k, \theta) &= y(k) - \hat{y}(k|\theta) \end{aligned} \quad (4)$$

where the function $V_N(\theta)$ denotes the loss which results from the model in the fitting process; ‘‘arg min’’ stands for the minimization argument of the function $V_N(\theta)$; N is the number of samples; and $\epsilon(k, \theta)$ represents the residuals. This requires an iterative search for θ that yields the minimum of the loss function $V_N(\theta)$. Equation (4) represents a nonlinear optimization problem, and thus may lead to local minima.

In this work, only the coefficients of polynomial $A(q)$ in (1) are of interest, since it is aimed at extracting the modal content of the signal, which are the roots of $A(q)$. Hence, one may consider applying techniques such as the Yule-Walker method that only estimates the parameters of $A(q)$, thus employing more simplified and robust numerical techniques. It is also possible to model $y[k]$ in (3) with a high-order AR model, rather than using an ARMA model, as a result of Kolmogorov’s theorem. This leads to an objective function that can be solved by means of well-known least square methods [12]. A high-order AR model, however, leads to extraneous modes close to the system modes, which could be difficult to distinguish from the true modes. Furthermore, an AR model may result in biased estimates if residuals are not white. It is also important to mention that when the signal-to-noise ratio (SNR) is low, the model structure is an ARMA rather than a pure AR. The least square modified Yule-Walker method, which has been employed in [6] to extract the modal content of the ambient noise, presents superior performance compared to the original Yule-Walker method as reported in [13].

The theory of variance of identified model parameters is well-understood and developed in the context of PEMs [9]. Hence, this theory, as explained below, is used here to estimate the variance of identified modes.

At the solution point $\hat{\theta}$, the differentiation of $V_N(\theta)$ with respect to θ has to be zero, i.e.

$$V'_N(\hat{\theta}) = 0 \quad (5)$$

Thus, an iterative algorithm, such as Newton, is used to solve for $\hat{\theta}$ by means of the Taylor series expansion of (5) around a given point θ^* close to $\hat{\theta}$ [11]:

$$0 \approx V'_N(\theta^*) + V''_N(\theta^*)(\hat{\theta} - \theta^*) \quad (6)$$

or

$$(\hat{\theta} - \theta^*) = -[V''_N(\theta^*)]^{-1} V'_N(\theta^*) \quad (7)$$

This requires the first derivative (gradient) and second derivative (Hessian); thus:

$$V'_N(\theta^*) = -\frac{1}{N} \sum_{k=1}^N \psi(k, \theta^*) \epsilon(k, \theta^*) \quad (8)$$

$$\begin{aligned} V''_N(\theta^*) &= \frac{1}{N} \sum_{k=1}^N \psi(k, \theta^*) \psi^T(k, \theta^*) + \\ &\frac{1}{N} \sum_{k=1}^N \psi'(k, \theta^*) \epsilon(k, \theta^*) \end{aligned} \quad (9)$$

where

$$\psi(k, \theta^*) = -\frac{d}{d\theta} \epsilon(k, \theta) \Big|_{\theta^*} = \frac{d}{d\theta} \hat{y}(k|\theta) \Big|_{\theta^*} \quad (10)$$

Close to the solution $\hat{\theta}$, the predicted errors $\epsilon(k, \theta)$ are independent; thus,

$$V''_N(\theta^*) \approx \frac{1}{N} \sum_{k=1}^N \psi(k, \theta^*) \psi^T(k, \theta^*) \quad (11)$$

A. Covariance of Parameters

It is known that $\sqrt{N}(\hat{\theta} - \theta^*)$ is asymptotically Gaussian distributed with zero mean and a covariance matrix P , i.e. $\mathcal{N}(0, P)$ [9]. Therefore, an estimate of P from available data can be obtained as follows:

$$\begin{aligned} \hat{P} &= \hat{\lambda}_0 \left(V''_N(\theta^*) \right)^{-1} \\ \hat{\lambda}_0 &= \frac{1}{N} \sum_{k=1}^N \epsilon^2(k, \theta^*) \end{aligned} \quad (12)$$

where $\hat{\lambda}_0$ is an estimate of the variance of the errors. Then, the covariance of parameter estimates, i.e. $P_{\hat{\theta}} = E[(\hat{\theta} - \theta^*)(\hat{\theta} - \theta^*)^T]$, can be approximated as:

$$P_{\hat{\theta}} \approx \frac{1}{N} \hat{P} \quad (13)$$

The modes of a system are the roots of the characteristic equation, and hence are only dependent on the AR part of an ARMA(p,d). Thus, the covariance matrix $P_{\hat{\theta}}$ is partitioned so

that the rows and columns corresponding to the AR and the MA parts are separate as follows:

$$P_{\hat{\theta}} = \begin{bmatrix} P_{\hat{\theta}_{AR}} & P_{\hat{\theta}_{ARMA}} \\ P_{\hat{\theta}_{ARMA}} & P_{\hat{\theta}_{MA}} \end{bmatrix} \quad (14)$$

A relationship between $P_{\hat{\theta}_{AR}}$ and the covariance of modes is established below.

B. Covariance of Modes

System modes can be related to θ_{AR} , which are the coefficients of the characteristic equation, as follows:

$$\Phi = \gamma(\theta_{AR}) \quad (15)$$

where $\gamma(\theta_{AR})$ is a nonlinear function, and Φ denotes a vector containing the modal parameters. For instance, the real part α and the frequency f of the modes can be used to define:

$$\Phi = [\alpha_1, f_1, \alpha_2, f_2, \dots, \alpha_p, f_p]^T \in \mathfrak{R}^{2p} \quad (16)$$

In order to obtain the mean and variance of the modes, the expected value operator may be applied to a Taylor series expansion of the function γ about an operating point $(\hat{\Phi}, \hat{\theta}_{AR})$; thus:

$$\Phi \approx \hat{\Phi} + J(\hat{\theta}_{AR}) (\theta_{AR} - \hat{\theta}_{AR}) \quad (17)$$

$$(18)$$

where

$$J(\hat{\theta}_{AR}) = \left. \frac{\partial \gamma(\theta_{AR})}{\partial \theta_{AR}} \right|_{\hat{\theta}_{AR}} \in \mathfrak{R}^{2p \times p} \quad (19)$$

Rearranging (17) and applying the second moment operator (covariance) yields:

$$\begin{aligned} \text{Cov } \Phi &= E [(\Phi - \hat{\Phi}) (\Phi - \hat{\Phi})^T] \\ &= J(\hat{\theta}_{AR}) P_{\hat{\theta}_{AR}} J^T(\hat{\theta}_{AR}) \end{aligned} \quad (20)$$

This clearly shows the connection between the covariance of estimates $P(\hat{\theta}_{AR})$ and the covariance of modes $\text{Cov } \Phi$. Therefore, in (20), $P_{\hat{\theta}_{AR}}$ can be estimated using (13), and a numeric Jacobian $J(\hat{\theta}_{AR})$ can be approximated as follows:

$$J_{ij}(\hat{\theta}_{AR}) \approx \frac{\gamma_i(\hat{\theta}_{AR} + \Delta\theta_j) - \gamma_i(\hat{\theta}_{AR} - \Delta\theta_j)}{2h} \quad (21)$$

where $\Delta\theta_j = [0 \ \dots \ 0 \ \underbrace{h}_j \ 0 \ \dots \ 0]$, with h being a small number.

III. TEST CASES

The proposed method for estimating the standard deviation of the identified modes is tested with the 2-area benchmark system and the IEEE 14-bus system. First, a Monte-Carlo type of simulation with 150 independent simulations is performed. For these trials, 1% of the loads are represented as Gaussian noise; 4-minutes data blocks of a generator output power are recorded in each simulation, and white Gaussian noise is added to the output signals as measurement noise, so that the SNR is 20 db. The signals are then passed through a Chebyshev low-pass filter with a cut-off frequency of 2 Hz, and resampled at 10 Hz rate. The preprocessed data blocks along with a PEM are employed to estimate the parameters of an ARMA(p,d) model representing the power system transfer function.

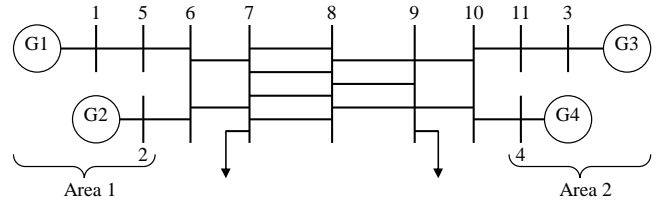


Fig. 1. Two-area benchmark system.

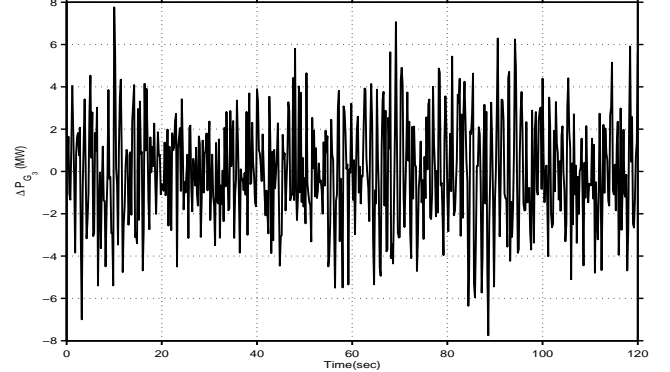


Fig. 2. Two-minute block measurement of the change in generator G_3 's power at 2734 MW loading level.

A. Two-area Benchmark System

A single line diagram of the system is shown in Fig. 1 [14]. The generators are modeled using subtransient models and simple exciters equipped with PSSs. The corresponding static and dynamic data is given in [13]. The total base loading level is 2734 MW and 200 MVar, and loads are modeled as constant PQ loads.

Areas 1 and 2 are connected through tie-lines, and an inter-area mode with a frequency of about 0.75 Hz is observed. The individual machines in each area also contribute to a local mode in the same area with frequencies of about 1.2 Hz and 1.4 Hz in Areas 1 and 2, respectively. Thus, an inter-area rotor angle mode and two local modes are observed for this test case. Figure 2 shows typical ambient noise on the generator G_3 's output power P_{G_3} . The power spectrum density of the P_{G_3} , obtained via an averaged modified periodogram Welch method [15], is depicted in Fig. 3, showing that both the inter-area and the local modes in Area 2 are being excited due to underlying, random load switching in the system.

An ARMA(p,d) model with different p 's and d 's is employed to model the measured signal in every simulation. The mean of the estimated electromechanical mode corresponding to each model for 150 trials is depicted in Fig. 4, together with the "true" mode, $-0.1228 \pm j4.7824$, obtained from a linearized model (LM) of the power system. Observe that when a pure AR(15) model, i.e. ARMA(15,0), is selected, the results are not as close to the LM mode as ARMA(p,d) with $d \neq 0$; this was also noticed in [6], [10]. From the system identification point of view, an ARMA model set is more likely to adequately represent the true system than an AR model; in this case, the estimated parameters would be asymptotically

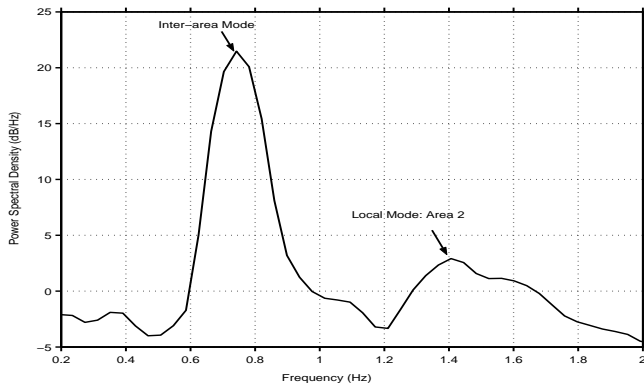


Fig. 3. Power spectral density of the ΔP_{G_3} in the 2-area benchmark system.

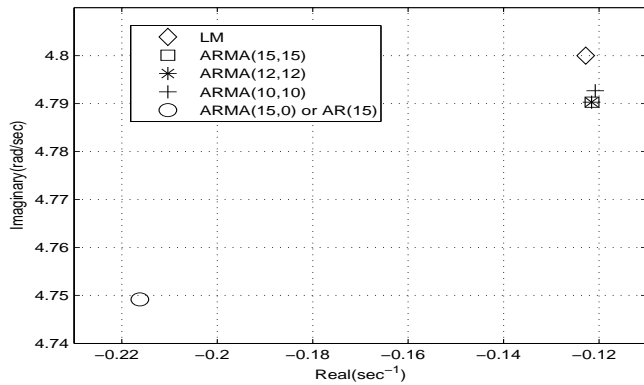


Fig. 4. Mean of the identified inter-area mode $-0.1228 \pm j4.7824$ for the 2-area benchmark system using a Monte-Carlo with 150 independent simulations.

unbiased [9], as observed in Figs. 5 and 6.

The estimated standard deviation of both the real part and frequency of the inter-area mode obtained using (20) is depicted in Figs. 7 and 8. Observe that the estimates track the results corresponding to the Monte-Carlo simulation, and the mean of the estimates can provide reasonably good accuracy with a significantly reduced number of trials. For instance, in Fig. 8, the convergence speed of the standard deviation of estimates is nearly 3 to 4 times faster than the Monte-Carlo method, thus yielding significant reduction in the monitoring time.

Notice that the uncertainty associated with the real part of the mode is relatively large when compared with the one for the frequency (e.g. the standard deviation of the real part of the mode depicted in Fig. 8 is about 25% of the actual real part, whereas it is only about 0.6% for the frequency). This is due to the fact that obtaining accurate estimates of mode damping in power systems using system identification is more difficult [5], [6], [7].

B. IEEE 14-bus System

This test system is shown in Fig. 9. It has 5 generators with two of them providing both active and reactive power at Buses 1 and 2; the generators at Buses 3, 6 and 8 are basically synchronous condensers [16]. The generators are modeled by

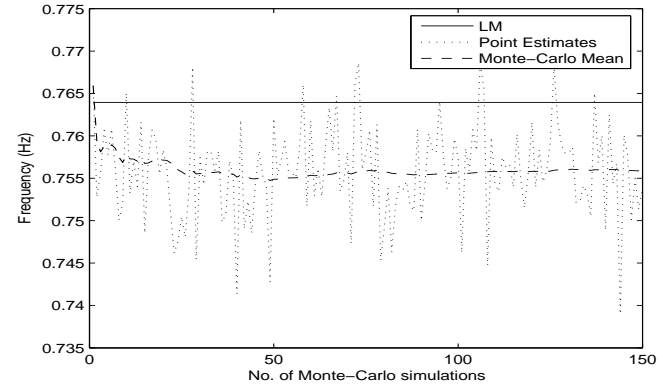
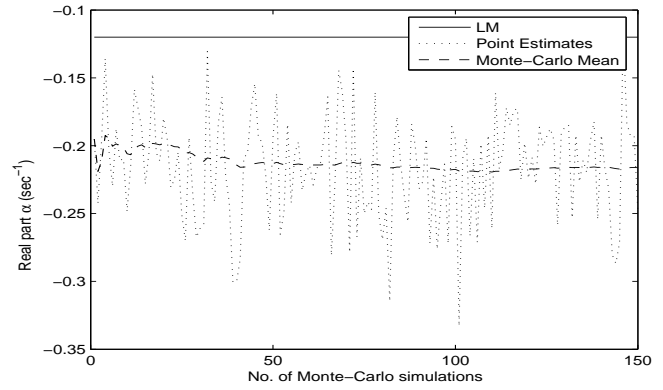


Fig. 5. Real part and frequency of the identified inter-area mode $-0.1228 \pm j4.7824$ for the 2-area benchmark system; AR(15).

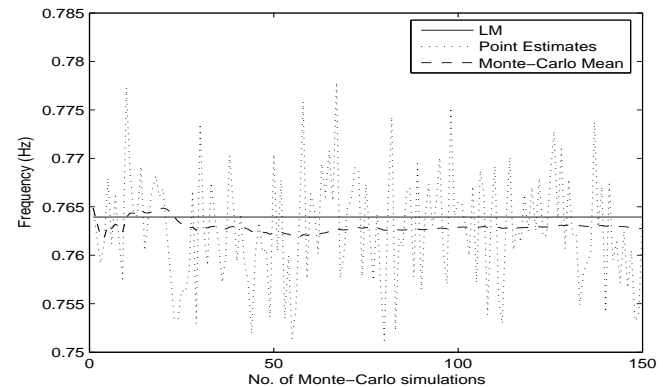
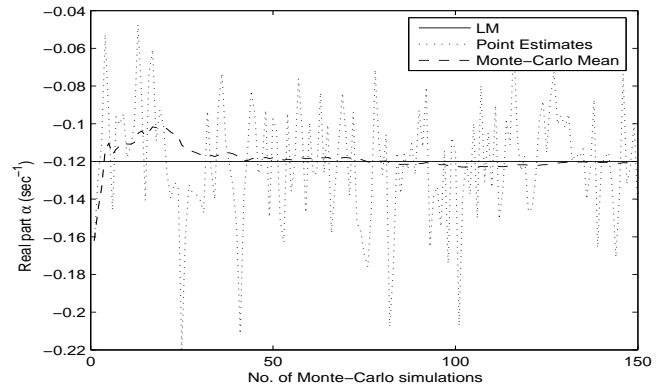


Fig. 6. Real part and frequency of the identified inter-area mode $-0.1228 \pm j4.7824$ for the 2-area benchmark system; ARMA(10,10).

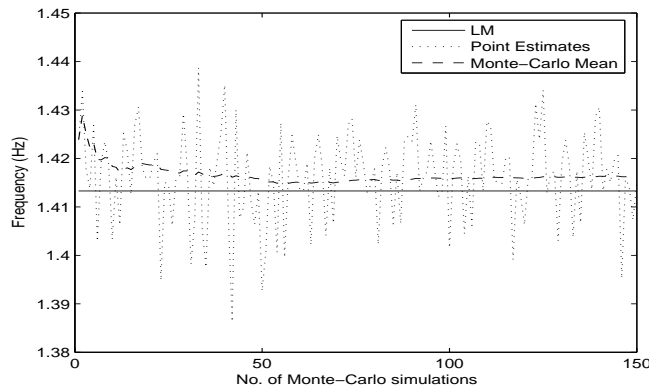
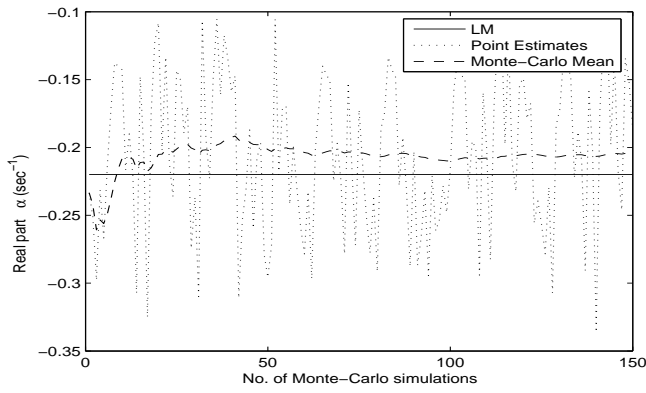


Fig. 10. Real part and frequency of the identified inter-area mode $-0.22 \pm j8.83$ for the IEEE 14-bus system; ARMA(12,12).

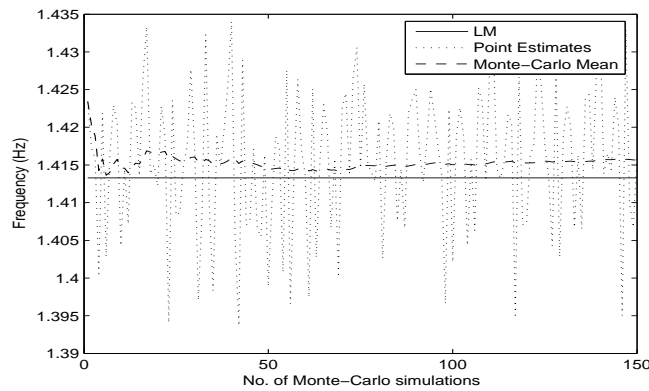
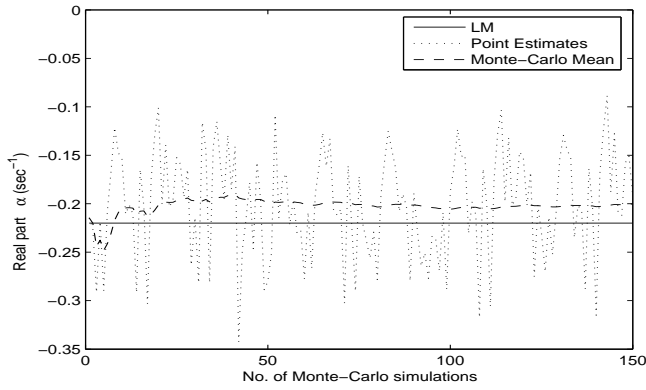


Fig. 11. Real part and frequency of the identified inter-area mode $-0.22 \pm j8.83$ for the IEEE 14-bus system; ARMA(14,14).

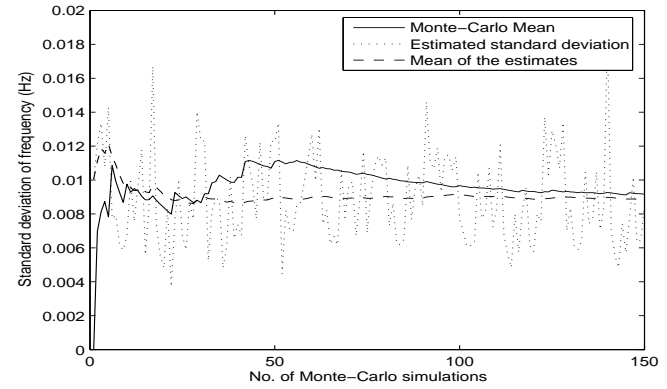
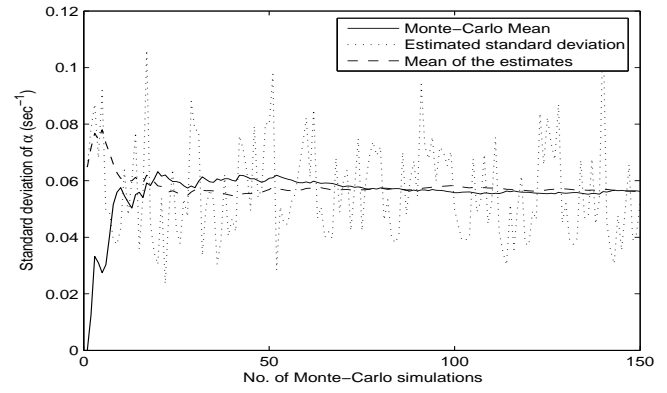


Fig. 12. Standard deviation of the real part and the frequency of the identified electromechanical mode $-0.22 \pm j8.83$ for the IEEE 14-bus system using Monte-Carlo and equation (20); ARMA(12,12).

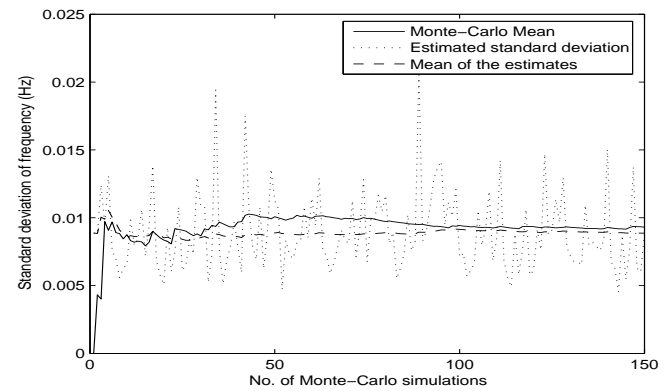
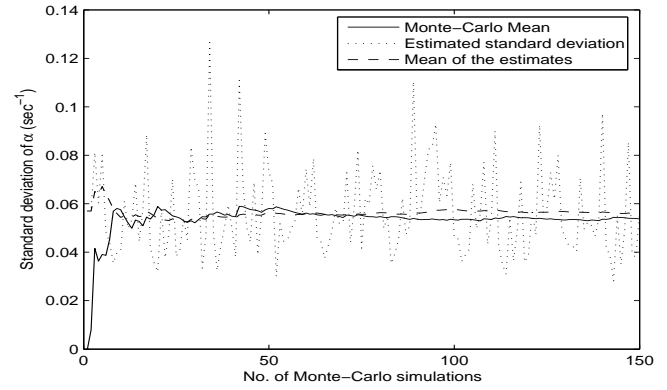


Fig. 13. Standard deviation of the real part and the frequency of the identified electromechanical mode $-0.22 \pm j8.83$ for the IEEE 14-bus system using Monte-Carlo and equation (20); ARMA(14,14).

IV. CONCLUSIONS

A novel procedure to calculate the second order statistical properties of identified electromechanical modes from ambient noise in power systems is presented and justified. The proposed method is based on a technique that uses Taylor series expansions to establish a connection between the variance of model parameters and the variance of eigenvalues. The variance of parameters are estimated using only one data block, i.e. one set of measurements, and this information is then employed to estimate the uncertainty associated with the identified electromechanical modes. The proposed methodology was tested using measurements obtained from two benchmark systems; the results obtained demonstrate the accuracy and possible advantages of the proposed method for on-line modal analysis applications.

The proposed technique can be used to avoid Monte-Carlo type of analyses, thus resulting in a significant reduction in computational time. This method should facilitate the use of ambient noise in on-line modal analysis applications, such as system control or real-time security monitoring, since it does not require any artificial disturbances such as adding/tripping generators.

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