Cournot and Affine Supply Function Equilibria in Integrated Electricity Markets

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Abstract

A nonlinear programming (NLP) formulation to study Nash equilibria in electricity markets is proposed. Cournot and affine supply function equilibria (ASFE) are formulated and compared. The market model relies on a DC network with a quadratic approximation for losses. The formulation of the competition is set as an equilibrium problem using a multi-leader single-follower game. The problem is then reformulated as a standard NLP problem. This formulation avoids the use of diagonalization or other iterative computational schemes to attain an equilibrium. A 14-node system is used for the numerical comparison of approaches. It is found that both the Cournot and ASFE approaches attained the same equilibrium.

Key words: Electricity Markets, Nash Equilibria, Cournot, Affine Supply Function, Energy, Spinning Reserves, Market Power.

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1 Introduction

In power systems where competition has been introduced, the activities are primarily driven by business-oriented incentives. A power market can be based upon the minimization of the social cost, which is equivalent to maximizing the difference between consumer surplus and supplier cost. Suppliers and consumers submit their respective bids, and an independent market operator is in charge of computing a market equilibrium. The main behavioural assumption to analyze competition is that both suppliers and consumers act to maximize their profits. For a competitive power market, the main assumption is that suppliers cannot affect the market price, and, consequently, they will bid their true marginal costs. In contrast, in a monopoly there is one dominant supplier that can profitably manipulate the market outcome; that is, the supplier has market power. By definition, market power is the ability to profitably set prices above competitive levels [1]. However, not all high market prices are the result of exercising market power; some high prices are only a product of scarce generation [2]. Market power can arise as one of the most critical imperfections of electricity markets, and has become one of the main concerns in the design and monitoring of markets [3].

There is a large variety of strategic behaviour models that vary in: i) the system elements taken into account; ii) the mechanism for pricing; iii) the way the transactions are made; and iv) the strategies of the Generation Companies (GenCos). Depending on how the transactions are made, two models can be implemented: bilateral-based markets and pool-like markets. In this paper, a pool-like model is considered. This kind of market is characterized by a central pricing mechanism for transmission, which is carried out by an Independent

System Operator (ISO), and participants buy and sell energy only through the ISO. Games can be classified in two kinds: i) symmetrical games, such as Bertrand (game in prices), Cournot (game in quantities) and affine Supply Function Equilibrium (game in bids); and ii) asymmetrical games, such as the Stackelberg, which can also be in quantities or bids. The kind of game defines how competition occurs among market participants.

Currently, imperfect competition in electricity markets is widely modelled as equilibrium problems [4–7]. Equilibrium problems capture the multi-agent perspective. In addition, advances in complementarity theory provide a better understanding of some aspects of equilibria. An equilibrium problem is defined by a set of optimization problems, one per market participant (suppliers, consumers and market operator), which relate prices, generation levels, demands and power flows to satisfy every market participant's first-order optimality conditions. If a solution exists for the equilibrium problem, then no market participant will unilaterally alter its current position [8].

Hobbs [5] introduces a linear complementarity approach for modelling imperfect competition in both the pool and bilateral markets. He uses a congestion pricing scheme to allocate the transmission to participants. Most DC models of imperfect competition have neglected the impact of losses, as it violates linearity. Disregarding losses may lead to biased market outcomes, and thus to misleading conclusions about market power [9]. In [10], a multi-period model for oligopolistic markets is presented; however, this model does not explore the impact of losses on the bidding strategies and market outcomes. In [7], losses are accurately computed using a detailed AC transmission system, which leads to a complex nonconvex formulation.

The contribution of this paper to the analysis of competition is twofold: i) a nonlinear formulation of the game using a DC approximation of the network with a quadratic approximation of losses; and ii) a *vis-à-vis* study of the Cournot and ASFE approaches. This paper is organized as follows. In Section 2, the main elements of the models are described. The formulations of the Cournot and the ASFE games are introduced in Sections 3 and 4, respectively. The NLP formulations of the games are derived in Section 5. Computational results are reported in Section 6. Conclusions are given in Section 7.

2 Market Formulation

For the study of competition in this paper, a pool-like market is used where energy and spinning reserves are jointly optimized. There is a central entity in charge of clearing the joint energy and reserves market, and of managing the congestion simultaneously. Both generation companies (GenCos) and demands are required to submit bids to trade power. Then the ISO runs an optimal power flow (OPF) to determine the optimal schedule and the locational marginal prices (LMPs) at which power will be traded. These LMPs comprise the information of not only the generation cost, but also of the value of the transmission when it becomes scarce.

2.1 Generation Companies

The generation cost of unit h placed at node i is represented by a nondecreasing convex function, i.e.,

$$c_{h,i}(g_{h,i}) = \beta_{h,i} g_{,h,i} + \gamma_{h,i} g_{h,i}^2, \tag{1}$$

where $\beta_{h,i} \in \mathbb{R}$ and $\gamma_{h,i} \in \mathbb{R}_+$ are the coefficients of the true cost function, and $g_{h,i}$ is the generation level of the unit. The corresponding marginal cost function is

$$c'_{h,i}(g_{h,i}) = \beta_{h,i} + 2\gamma_{h,i}g_{h,i}. \tag{2}$$

This affine function can be seen as the inverse of the true supply bid.

Some generators can also provide spinning reserves (SR). The cost of providing SR can be from opportunity, variable and capital costs. The opportunity cost for providing SR is defined as the difference between the energy price and the marginal cost at the operating point of the unit. Depending on this operating point, this opportunity cost can sometimes be negligible [11]. In this paper, the SR cost is modelled using a linear function of the form

$$c^r_{h,i}(r_{h,i}) = \alpha_{h,i}r_{h,i}. \tag{3}$$

Also, upper and lower limits are considered. Without loss of generality, in this formulation all generation units have a lower limit of zero.

It is also assumed that only some GenCos can influence the market and behave strategically, and are referred to as *leaders*. An index ν is used to denote such GenCos, and the set of leaders is denoted by $\mathcal{V}$. In order to differentiate the output of these leaders from the output of the fringe of competitive generators, the symbols $X_{\nu,h,i}$ and $X^r_{\nu,h,i}$ stand for the active and spinning reserve outputs for unit h, placed at node i and controlled by GenCo ν.

The functions for the true generation and spinning reserves costs for leaders are of the same kind as those described for the competitive units, see *supra*. This will also hold for the corresponding generation limits to be used. In a pool-like market, however, leaders can submit functions that are not necessarily their true cost function. In order to denote the submitted function, the following apparent-cost expression is used:

$$\hat{c}_{\nu,h,i}(X_{\nu,h,i}) = \kappa^{\beta}_{\nu,h,i} X_{\nu,h,i} + \kappa^{\gamma}_{\nu,h,i} X^2_{\nu,h,i}, \tag{4}$$

where $\kappa^{\beta}_{\nu,h,i} \in \mathbb{R}$ and $\kappa^{\gamma}_{\nu,h,i} \in \mathbb{R}_+$ are (linear and quadratic) coefficients. The supply bid function will be the marginal of the apparent-cost function with a similar structure to that of the true bid function, i.e.,

$$\hat{c}'_{\nu,h,i}(X_{\nu,h,i}) = \kappa^{\beta}_{\nu,h,i} + 2\kappa^{\gamma}_{\nu,h,i} X_{\nu,h,i}. \tag{5}$$

Similarly, the bid function for SR is

$$\hat{c}^{r}_{\nu,h,i}(X^{r}_{\nu,h,i}) = \kappa^{\alpha}_{\nu,h,i} X^{r}_{\nu,h,i}. \tag{6}$$

2.2 Demands

The benefit of the demand placed at node i is represented by the nondecreasing concave function

$$b_i(d_i) = \rho_i d_i - \delta_i d_i^2, \tag{7}$$

where ρ_i and δ_i are coefficients, and d_i is the demand level. The marginal benefit function provides the inverse demand function

$$b'_i(d_i) = \rho_i - 2\delta_i d_i. \tag{8}$$

This function denotes a price-responsive demand. In addition to the demand for active power, there is a requirement for SR to be fulfilled. Usually, the amount of SR is set according to the level of active demand. Following [6], the SR requirement is set as a percentage ϵ of the net system's demand, i.e.,

$$\overline{R} = \epsilon \sum_{i} d_i. \tag{9}$$

2.3 Transmission System

A typical simplification of AC transmission networks is the linearization of the power flow expressions. This linearization relies on three key assumptions: i) the resistance of transmission lines is small compared to their reactance, so that the former can be neglected; ii) voltage (or nodal) angles as well as differences in voltage angles are assumed to be reasonably small; and iii) there is sufficient reactive power compensation at all nodes to keep voltage levels constant at desirable levels, say, at one per unit value.

With these assumptions, expressions and terms related to the reactive power flow are dropped, and the active power flow expressions are reduced to a set of linear relationships in terms of voltage angles. Due to these assumptions, the linear approximation is suitable to calculate active power flows and voltage angles, but it does not provide any insight regarding voltage magnitudes and reactive power flows. Nonetheless, this approximation relies on the fact that there exists a strong coupling between active power and voltage angle, and reactive power and voltage magnitude. This feature is what validates the analysis of active power without explicit consideration of reactive power.

Let us consider a power network where the finite set of nodes is denoted by $\mathcal{N}$, and N_i stands for the set of nodes directly connected to node i. The set of transmission lines is denoted by $\mathcal{L}$. The flow z_{ij} in a transmission line between nodes i and j is given by

$$z_{ij} = B_{ij}(\theta_i - \theta_j), \tag{10}$$

where $B_{ij} = x_{ij}/(r_{ij}^2 + x_{ij}^2)$, and r_{ij} and x_{ij} are the resistance and inductance of the transmission line. The symbol θ_i is used to denote the voltage angle at node i. The symbol $\overline{z}_{ij}$ is used to denote the maximum power flow in MW for transmission line ij in either direction.

Also, the losses in a transmission line can be approximated by a quadratic formula in terms of the nodal angles [12],

$$\frac{1}{2}G_{ij}(\theta_i - \theta_j)^2, \tag{11}$$

where $G_{ij} = r_{ij}/(r_{ij}^2 + x_{ij}^2)$. Within the market formulation, the losses of each transmission line are modelled as an artificial demand equal to half of the losses at each node to which the line is connected.

2.4 Kinds of Games

Two kinds of games are studied in this paper: the Cournot and the ASFE games. In the Cournot game, the strategic decisions of generators (leaders) are the generation and spinning reserve outputs, i.e., it is a game among leaders in terms of quantities. On the other hand, in the ASFE game, the strategic variables of leaders are the coefficients of both the energy and SR bids; i.e., it is a game among leaders in terms of bids. Moreover, there is a game between each leader and the competitive fringe of generators. This is well known as a leader-follower or Stackelberg game.

3 Cournot Game

In the (noncooperative) Cournot case, every market participant maximizes its profits considering an expectation of the output power of the other firms, i.e., each GenCo conjectures that if it varies its generation level, rivals will hold their output fixed. A point to be highlighted is that a Cournot equilibrium is also a Nash equilibrium: every supplier gets its best status given that the output of the other suppliers is fixed.

3.1 ISO problem

For the leader-follower game, the competitive fringe is composed not only of all the competitive generators, but it also includes the demand (assumed to be competitive). In this way, the central operator (ISO) can be viewed as the entity that characterizes a single follower. That is, the ISO clears the market taking as given the decisions of the leaders because it cannot alter them, and dispatching the competitive units. Hence, the follower's problem can be cast as a classical OPF:

$$\min \quad \sum_{f,i} \left\{ \beta_{f,i} g_{f,i} + \gamma_{f,i} g_{f,i}^2 \right\} - \sum_{j} \left\{ \rho_j d_j - \delta_j d_j^2 \right\} + \sum_{f,i} \alpha_{f,i} r_{f,i} \tag{12}$$

$$\text{s.t.} \quad \sum_{\nu,h} X^{\star}_{\nu,h,i} + \sum_{f} g_{f,i} - d_i - \sum_{j \in \mathcal{N}_i} \Big\{ B_{ij}(\theta_i - \theta_j) + \frac{1}{2} G_{ij}(\theta_i - \theta_j)^2 \Big\} = 0, \quad [\lambda_i], \quad \forall i \in \mathcal{I}, \tag{13}$$

$$\sum_{\nu,h} X^{r\star}_{\nu,h,i} + \sum_{f,i} r_{f,i} \geq \overline{R}, \quad [\mu^r] \tag{14}$$

$$|B_{ij}(\theta_i - \theta_j)| \leq \overline{z}_{ij}, \quad [\overline{\mu}_{ij}, \underline{\mu}_{ij}], \quad \forall ij \in \mathcal{L}, \tag{15}$$

$$g_{f,i} + r_{f,i} \leq \overline{g}_{f,i}, \quad [\overline{\mu}^g_{f,i}], \quad \forall f \in \mathcal{F}, i \in \mathcal{I}, \tag{16}$$

$$r_{f,i} \leq \overline{r}_{f,i}, \quad [\overline{\mu}^r_{f,i}], \quad \forall f \in \mathcal{F}, i \in \mathcal{I}, \tag{17}$$

$$d_i, \, g_{f,i}, r_{f,i} \geq 0. \tag{18}$$

The objective is to minimize the social cost defined as the difference between the cost of generation and SR, and the benefits of demand. Expression (13) stands for the power balance at each node of the system. The first term is the generation provided by the leaders. These generations are denoted with a symbol $\star$ because from the point of view of the ISO they are not variables, as they are exogenous. From the point of view of each leader, however, as well as from the point of view of the equilibrium problem, they are variables. The second summation accounts for the generation provided by the competitive fringe. Expression (13) also has the transmission component of power flows, and an artificial demand to account for one half of the losses of all transmission lines connected to node i. Expression (14) stands for the SR requirements. Expression (15) stands for the transmission line limits in either direction. Expressions (16) and (17) are the upper limits for generation and spinning reserves. On the right-hand side of each constraint, the associated dual variables are introduced in square brackets. For simplicity in the formulation, the OPF problem (12)-(18) can be cast as a standard NLP problem

$$\min \quad b(\boldsymbol{w}_0) \tag{19}$$

$$\text{s.t.} \quad \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{x}, \boldsymbol{w}_0,\, \boldsymbol{w}_1) = \boldsymbol{0}, \quad : [\boldsymbol{\lambda}], \tag{20}$$

$$\boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_0,\, \boldsymbol{w}_1) \geq \boldsymbol{0}, \quad : [\boldsymbol{\mu}], \tag{21}$$

$$\boldsymbol{w}_0 \geq \boldsymbol{0}, \tag{22}$$

where equality and inequality constraints are comprised into $\boldsymbol{c}_{\mathcal{E}}(\boldsymbol{x}, \boldsymbol{w}_0,\, \boldsymbol{w}_1)$ and $\boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_1)$ respectively, while their corresponding dual variables are comprised into vectors $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. The vectors $\boldsymbol{w}_0$ and $\boldsymbol{w}_1$ contain the control and state variables of the ISO problem such that $\boldsymbol{w}_0$=($\boldsymbol{d}$, $\boldsymbol{g}$, $\boldsymbol{r}$) and $\boldsymbol{w}_1$=($\boldsymbol{\theta}$). The vector $\boldsymbol{x} = (\boldsymbol{X},\, \boldsymbol{X}^r)$ contains all the decisions variables of leaders. For the sake of simplicity, all the function arguments are omitted. This formulation yields the following compact form of the first-order optimality conditions of the ISO problem:

$$\mathbf{0} = \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{E}} \boldsymbol{\lambda} - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{I}} \boldsymbol{\mu}, \tag{23}$$

$$\mathbf{0} = \boldsymbol{c}_{\mathcal{E}}, \tag{24}$$

$$\mathbf{0} \leq \nabla_{\boldsymbol{w}_0} b - \nabla^T_{\boldsymbol{w}_0} \boldsymbol{c}_{\mathcal{E}} \boldsymbol{\lambda} - \nabla^T_{\boldsymbol{w}_0} \boldsymbol{c}_{\mathcal{I}} \boldsymbol{\mu} \quad \perp \quad \boldsymbol{w}_0 \geq \mathbf{0}, \tag{25}$$

$$\mathbf{0} \leq \boldsymbol{c}_{\mathcal{I}} \quad \perp \quad \boldsymbol{\mu} \geq \mathbf{0}, \tag{26}$$

$$\boldsymbol{\lambda} \quad \text{free.} \tag{27}$$

The symbol $\perp$ is used throughout the paper to denote a complementarity condition arising from inequality constraints. By defining the variable vectors $\boldsymbol{y}_0=(\boldsymbol{w}_0,\, \boldsymbol{\mu})$ and $\boldsymbol{y}_1=(\boldsymbol{w}_1,\, \boldsymbol{\lambda})$, the expressions (23) and (24) can be comprised into a function $\boldsymbol{h}_{\mathcal{E}}(\boldsymbol{x},\, \boldsymbol{y}_0,\, \boldsymbol{y}_1) = \mathbf{0}$ and expressions (25) and (26) can be comprised into a function $\boldsymbol{h}_{\mathcal{I}}(\boldsymbol{y}_0,\, \boldsymbol{y}_1) \geq \mathbf{0}$.

By introducing slack variables $\boldsymbol{s}$ into the latter function, the KKT conditions of the ISO can be stated as

$$\mathbf{0} = \boldsymbol{h}_{\mathcal{E}}(\boldsymbol{x},\, \boldsymbol{y}_0,\, \boldsymbol{y}_1), \tag{28}$$

$$\mathbf{0} = \boldsymbol{h}_{\mathcal{I}}(\boldsymbol{y}_0,\, \boldsymbol{y}_1) - \boldsymbol{s}, \tag{29}$$

$$\mathbf{0} \leq \boldsymbol{s} \quad \perp \quad \boldsymbol{y}_0 \geq \mathbf{0}. \tag{30}$$

This set of expressions gives the stationarity conditions for the primal ISO problem.

3.2 Leaders Problem

Each GenCo that behaves strategically is considered as a leader within the game formulation. The leaders can profit from participating in both the energy and SR markets. The profit function of leader ν is

$$\begin{aligned}\Pi_\nu = f_\nu(\boldsymbol{x}_\nu, \boldsymbol{y}_0, \boldsymbol{y}_1) = &\sum_{h,i} \Big\{ \lambda_i X_{\nu,h,i} - c_{\nu,h,i}(X_{\nu,h,i}) \Big\} + \\ &\sum_{h,i} \Big\{ (\mu_i^r - \alpha_{\nu,h,i}) X^r_{\nu,h,i} \Big\}. \end{aligned} \tag{31}$$

The first summation is the profit from the energy market (revenues minus costs), and the second summation is the profit from SR. The set of leaders is given by $\mathcal{V}$, and their decision variables form a vector $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_{|\mathcal{V}|})$. Thus, given leader ν, its rivals' decisions $\{\boldsymbol{x}_\ell | \ell \neq \nu\}$ are denoted as $\boldsymbol{x}_{-\nu}$; this vector is considered to be fixed within each leader problem. This means that each leader problem is parametrized by $\boldsymbol{x}_{-\nu}$. Because market prices come from the ISO problem and are common to all leaders, such prices can be seen as shared decision variables. Since the leaders' profit depends on both its own decisions and the market prices, the profit function is denoted as $f(\boldsymbol{x}_\nu, \boldsymbol{y}_1)$. Thus, the leader can be characterized by the following profit-maximization problem:

$$\min \quad - f_\nu(\boldsymbol{x}_\nu, \boldsymbol{y}_1) \tag{32}$$

$$\text{s.t.} \quad \boldsymbol{h}_\nu(\boldsymbol{x}_\nu) \geq \boldsymbol{0}, \tag{33}$$

$$\boldsymbol{h}_{\mathcal{E}}(\boldsymbol{x}_\nu,\, \boldsymbol{y}_0,\, \boldsymbol{y}_1;\, \boldsymbol{x}_{-\nu}) = \boldsymbol{0}, \tag{34}$$

$$\boldsymbol{h}_{\mathcal{I}}(\boldsymbol{y}_0,\, \boldsymbol{y}_1) - \boldsymbol{s} = \boldsymbol{0}, \tag{35}$$

$$\boldsymbol{0} \leq \boldsymbol{y}_0 \quad \perp \quad \boldsymbol{s} \geq \boldsymbol{0}. \tag{36}$$

Since the game is à la Cournot, each leader takes its rivals' decisions as given.

This is applied in both generation and SR outputs. Expression (33) stands for upper limits of energy and SR for leader ν. The inclusion of the ISO stationarity conditions (34)-(36) implies that each leader anticipates how the ISO reacts to its decisions. Because of this inclusion, each leader problem is a Mathematical Problem with Complementarity Constraints (MPCC). The multi-leader setting is harder than the single-leader problem because all leaders share the complementarity constraints given by the KKT conditions of the ISO. Problems composed by a set of MPCCs linked through shared decisions variables are called Equilibrium Problems with Complementarity Constraints (EPCC).

4 ASFE Game

In the (noncooperative) ASFE case, every market participant maximizes its profits considering an expectation of the energy and SR bids of the other leaders, i.e., each GenCo conjectures that if it varies its bids, rivals will keep their bids unaltered. At this kind of Nash equilibrium, every supplier gets its best status given the bids of the other suppliers as fixed.

4.1 ISO problem

Similar to the Cournot case, the ISO problem characterizes the follower:

$$\min \quad \sum_{f,i} \left\{\beta_{f,i} g_{f,i} + \gamma_{f,i} g_{f,i}^2\right\} + \sum_{f,i} \alpha_{f,i} r_{f,i} \tag{37}$$

$$+ \sum_{\nu,h,i} \left\{\kappa_{f,i}^{\beta\star} X_{\nu,h,i} + \kappa_{\nu,h,i}^{\gamma\star} X_{\nu,h,i}^2\right\} + \sum_{\nu,h,i} \kappa_{\nu,h,i}^{\alpha\star} X_{\nu,h,i}^r\Big\} \tag{38}$$

$$- \sum_{j} \left\{\rho_i d_i - \delta_i d_i^2\right\} \tag{39}$$

$$\text{s.t.} \quad \sum_{\nu,h} X_{\nu,h,i} + \sum_{f} g_{f,i} - d_i - \sum_{j \in \mathcal{N}_i} \Big\{B_{ij}(\theta_i - \theta_j) +$$

$$\frac{1}{2} G_{ij}(\theta_i - \theta_j)^2\Big\} = 0, \quad [\lambda_i], \quad \forall i \in \mathcal{I} \tag{40}$$

$$\sum_{\nu,h} X_{\nu,h,i}^r + \sum_{f,i} r_{f,i} \geq \overline{R}, \tag{41}$$

$$|B_{ij}(\theta_i - \theta_j)| \leq \overline{z}_{ij}, \quad [\overline{\mu}_{ij}, \underline{\mu}_{ij}], \quad \forall ij \in \mathcal{L} \tag{42}$$

$$g_{f,i} + r_{f,i} \leq \overline{g}_{f,i}, \quad [\overline{\mu}_{f,i}^g], \quad \forall f \in \mathcal{F}, i \in \mathcal{I}, \tag{43}$$

$$r_{f,i} \leq \overline{r}_{f,i}, \quad [\overline{\mu}_{f,i}^r], \quad \forall f \in \mathcal{F}, i \in \mathcal{I}, \tag{44}$$

$$X_{\nu,h,i} + X_{\nu,h,i}^r \leq \overline{X}_{\nu,h,i}, \quad [\overline{\mu}_{\nu,h,i}^X], \quad \forall \nu \in \mathcal{V}, h \in \mathcal{H}, i \in \mathcal{I}, \tag{45}$$

$$X_{\nu,h,i}^r \leq \overline{X}_{\nu,h,i}^r, \quad [\overline{\mu}_{\nu,h,i}^{Xr}], \quad \forall \nu \in \mathcal{V}, h \in \mathcal{H}, i \in \mathcal{I}. \tag{46}$$

The minimization of the cost is also over the bids of the leaders. In this case, the ISO takes as given the bids of all suppliers (competitive ones submit their true cost function, while leaders submit their supply bid), and dispatches all the suppliers including the leaders. Thus, the generation and SR outputs of leaders are also the ISO's variables. From the point of view of the ISO, however, the coefficients of the bids are not variables but constants, and from the point of view of leaders, such coefficients are their strategic variables.

4.2 Leaders Problem

The leaders' decision variables $(\kappa^{\alpha}, \kappa^{\beta}, \kappa^{\gamma})$ form a vector $\boldsymbol{\varphi} = (\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, ..., \boldsymbol{\varphi}_{|\mathcal{V}|})$. Thus, given leader ν, its rivals' bids $\{\boldsymbol{\varphi}_\ell | \ell \neq \nu\}$ are denoted as $\boldsymbol{\varphi}_{-\nu}$; this vector is considered to be fixed within each leader problem. This means that each leader problem is parametrized by $\boldsymbol{\varphi}_{-\nu}$. On the other hand, the generation and SR outputs for leaders are considered similar to those of the competitive players and comprised into the vector $\boldsymbol{\omega}_0$. Following the derivation of the Cournot section, the profit function for each leader under an ASFE is

$$\begin{aligned} \Pi_\nu = f_\nu(\boldsymbol{y}_0, \boldsymbol{y}_1) = & \sum_{h,i} \Big\{ \lambda_i X_{\nu,h,i} - c_{\nu,h,i}(X_{\nu,h,i}) \Big\} + \\ & \sum_{h,i} \Big\{ (\mu_i^r - \alpha_{\nu,h,i}) X_{\nu,h,i}^r \cdot \Big\}. \end{aligned} \tag{47}$$

This function is the same as in the Cournot case. However, the leader's profit maximization problem becomes

$$\min \quad - f_\nu(\boldsymbol{y}_0, \boldsymbol{y}_1) \tag{48}$$

$$\text{s.t.} \quad \boldsymbol{h}_\nu(\boldsymbol{\varphi}_\nu) \geq \mathbf{0}, \tag{49}$$

$$\boldsymbol{h}_{\mathcal{E}}(\boldsymbol{\varphi}_\nu,\, \boldsymbol{y}_0,\, \boldsymbol{y}_1;\, \boldsymbol{\varphi}_{-\nu}) = \mathbf{0}, \tag{50}$$

$$\boldsymbol{h}_{\mathcal{I}}(\boldsymbol{y}_0,\, \boldsymbol{y}_1) - \boldsymbol{s} = \mathbf{0}, \tag{51}$$

$$\mathbf{0} \leq \boldsymbol{y}_0 \quad \perp \quad \boldsymbol{s} \geq \mathbf{0}. \tag{52}$$

Note that the leaders' problems for Cournot and ASFE are mathematically quite similar. The main difference lies in the kind of the decisions variables of the leaders in each approach.

5 Equilibrium Problem

For simplicity in the presentation, the following derivation is only presented for the Cournot approach. The derivation for the ASFE is similar. Recent formulations of EPCCs allow the game problem to be stated as a single NLP problem. Given the vast research done in NLP and the wide availability of NLP sofware, the resulting problem can be efficiently solved. For the NLP formulation of the game, we have followed the derivation proposed in [13,7]. Formulating complementarity constraint (36) as $\boldsymbol{y}_0 \circ \boldsymbol{s} \leq \mathbf{0}$,and using the notion of strong stationarity [14], the following stationarity conditions are obtained:

$$\mathbf{0} = \nabla_{\boldsymbol{x}_\nu} f - \nabla^T_{\boldsymbol{x}_\nu} \boldsymbol{h}_\nu \boldsymbol{\phi}_\nu - \nabla^T_{\boldsymbol{x}_\nu} \boldsymbol{h}_{\mathcal{E}_\nu} \overline{\boldsymbol{\vartheta}}_\nu, \qquad \forall \nu, \tag{53}$$

$$\mathbf{0} = \nabla_{\boldsymbol{y}_0} f - \nabla^T_{\boldsymbol{y}_0} \boldsymbol{h}_{\mathcal{E}_\nu} \overline{\boldsymbol{\vartheta}}_\nu - \nabla^T_{\boldsymbol{y}_0} \boldsymbol{h}_{\mathcal{I}_\nu} \underline{\boldsymbol{\vartheta}}_\nu + \boldsymbol{s} \circ \boldsymbol{\xi}_\nu - \boldsymbol{\psi}_\nu, \qquad \forall \nu, \tag{54}$$

$$\mathbf{0} = \nabla_{\boldsymbol{y}_1} f - \nabla^T_{\boldsymbol{y}_1} \boldsymbol{h}_{\mathcal{E}_\nu} \overline{\boldsymbol{\vartheta}}_\nu - \nabla^T_{\boldsymbol{y}_1} \boldsymbol{h}_{\mathcal{I}_\nu} \underline{\boldsymbol{\vartheta}}_\nu, \qquad \forall \nu, \tag{55}$$

$$\mathbf{0} = \underline{\boldsymbol{\vartheta}}_\nu + \boldsymbol{y}_0 \circ \boldsymbol{\xi}_\nu - \sigma_\nu, \qquad \forall \nu, \tag{56}$$

$$\mathbf{0} \leq \boldsymbol{h}_\nu \quad \perp \quad \boldsymbol{\phi}_\nu \geq \mathbf{0}, \qquad \forall \nu, \tag{57}$$

$$\mathbf{0} \leq \boldsymbol{y}_0 \quad \perp \quad \boldsymbol{\psi}_\nu \geq \mathbf{0}, \qquad \forall \nu, \tag{58}$$

$$\mathbf{0} \leq \boldsymbol{s} \quad \perp \quad \boldsymbol{\sigma}_\nu \geq \mathbf{0}, \qquad \forall \nu, \tag{59}$$

$$\mathbf{0} \leq -\boldsymbol{y}_0 \circ \boldsymbol{s} \quad \perp \quad \boldsymbol{\xi}_\nu \geq \mathbf{0}, \qquad \forall \nu, \tag{60}$$

$$\mathbf{0} = \boldsymbol{h}_{\mathcal{E}}, \tag{61}$$

$$\mathbf{0} = \boldsymbol{h}_{\mathcal{I}} - \boldsymbol{s}. \tag{62}$$

The initial complementarity condition of (36) can be recovered, and slack variables $\boldsymbol{\tau}_\nu$ are introduced in constraint (57) in order to have only equality constraints. All the complementarity conditions are formulated as NLP constraints of the form $\boldsymbol{a}^T\boldsymbol{b} \leq 0$ with $\boldsymbol{a}$, $\boldsymbol{b} \geq 0$, and all the terms of the form $\boldsymbol{a}^T\boldsymbol{b}$ are aggregated in an objective function to be minimized. This gives rise to the following NLP problem:

$$\min \sum_{\nu} \left\{ \boldsymbol{\phi}_\nu^T \boldsymbol{\tau}_\nu + \boldsymbol{\sigma}_\nu^T \boldsymbol{s} + \boldsymbol{\psi}_\nu^T \boldsymbol{y}_0 \right\} + \boldsymbol{y}_0^T \boldsymbol{s} \tag{63}$$

$$\text{s.t. } \boldsymbol{0} = \nabla_{\boldsymbol{x}_\nu} f - \nabla_{\boldsymbol{x}_\nu}^T \boldsymbol{h}_\nu \boldsymbol{\phi}_\nu - \nabla_{\boldsymbol{x}_\nu}^T \boldsymbol{h}_{\mathcal{E}_\nu} \overline{\boldsymbol{\vartheta}}_\nu, \quad \forall \nu, \tag{64}$$

$$\boldsymbol{0} = \nabla_{\boldsymbol{y}_0} f - \nabla_{\boldsymbol{y}_0}^T \boldsymbol{h}_{\mathcal{E}_\nu} \overline{\boldsymbol{\vartheta}}_\nu - \nabla_{\boldsymbol{y}_0}^T \boldsymbol{h}_{\mathcal{I}_\nu} \underline{\boldsymbol{\vartheta}}_\nu + \boldsymbol{s} \circ \boldsymbol{\xi}_\nu - \boldsymbol{\psi}_\nu, \quad \forall \nu, \tag{65}$$

$$\boldsymbol{0} = \nabla_{\boldsymbol{y}_1} f - \nabla_{\boldsymbol{y}_1}^T \boldsymbol{h}_{\mathcal{E}_\nu} \overline{\boldsymbol{\vartheta}}_\nu - \nabla_{\boldsymbol{y}_1}^T \boldsymbol{h}_{\mathcal{I}_\nu} \underline{\boldsymbol{\vartheta}}_\nu, \quad \forall \nu, \tag{66}$$

$$\boldsymbol{0} = \underline{\boldsymbol{\vartheta}}_\nu + \boldsymbol{y}_0 \circ \boldsymbol{\xi}_\nu - \boldsymbol{\sigma}_\nu, \quad \forall \nu, \tag{67}$$

$$\boldsymbol{0} = \boldsymbol{h}_\nu - \boldsymbol{\tau}_\nu, \quad \forall \nu, \tag{68}$$

$$\boldsymbol{0} = \boldsymbol{h}_{\mathcal{E}}, \tag{69}$$

$$\boldsymbol{0} = \boldsymbol{h}_{\mathcal{I}} - \boldsymbol{s}, \tag{70}$$

$$\boldsymbol{0} \leq \boldsymbol{\tau}_\nu,\ \boldsymbol{\phi}_\nu,\ \boldsymbol{\psi}_\nu,\ \boldsymbol{\sigma}_\nu,\ \boldsymbol{\xi}_\nu, \quad \forall \nu, \tag{71}$$

$$\boldsymbol{0} \leq \boldsymbol{y}_0,\ \boldsymbol{s}. \tag{72}$$

The NLP formulation (63)-(72) aims to provide a feasible solution to the EPCC by leading all complementarity conditions to zero while enforcing all the constraints of the original system. With no standing objective function to account for, there is no need to use a large penalty factor for the aggregated terms. An objective value of zero means that a feasible solution has been found for the EPCC. In our implementations, an objective function value of the order of 10^{-5} sufficed to yield a reasonable solution to the game problem.

6 Computational Results

The standard IEEE-based test power system of 14 nodes and 20 transmission lines is used to represent the transmission system [15]. Transmission lines limits have been scaled accordingly to induce congestion. Five generating units and eight demands are considered in the simulation; their data are provided in Tables 1 and 2. GenCos can participate in both the energy and SR markets.

GenCos placed at nodes 4 and 9 are considered the dominant players, and therefore are modelled as the leaders that exercise market power. The other GenCos and all demands behave competitively. The same test system setting is used to simulate both the Cournot and the ASFE approaches. The results of both approaches, as well the competitive outcome, are presented in Tables 3 and 4.

The results show that the equilibrium points attained by both approaches are very similar. For both leaders, generation levels decrease, even though this decrease is much less pronounced for GenCo 1. Profit from the energy market increases more markedly for GenCo 1. Furthermore, by gaming SR, both GenCos are now providing reserves and getting an extra profit. The prices of both energy and SR increase overall when gaming takes place, and consequently less demand is consumed.

An interesting feature of the strategic behaviour –for either Cournot or ASFE– is the fact that the leaders induce GenCo 4 to use all of its capacity. In the competitive case, GenCo 4 is providing 470.6 MW of energy and no SR. In the Cournot outcome, it increases generation up to 524.8 MW and also provides SR of 125.2 MW, which together amount to its maximum capacity of 650 MW.

Therefore, for GenCo 4 there is an opportunity cost between providing energy or SR. This fact drives the price of spinning reserves up to \$4.5/MW, which is the SR price of the next GenCo (14) in the merit order for provision of SR. The case of the strategic behaviour with an ASFE model is even more interesting. Again, the leaders induce GenCo 4 to use all of its capacity: 523 MW of energy plus 127 MW of spinning reserves. However, there is an infinite set of optimal dispatches that attain the same equilibrium point. The leaders identify that \$4.5/MWh is the maximum bid they can offer. Otherwise, GenCo 14 (with a cost of \$4.5/MWh) will provide the required SR at that price. So both leaders' optimal bid for spinning reserve is \$4.5/MWh. Nonetheless, when all suppliers bid for the spinning reserves market, the ISO will schedule 127 MW to GenCo 4, and the extra 58.5 MW to be scheduled to satisfy the SR requirements can be allocated in any proportion among GenCos 1, 9 and 14. This impacts the profit that each leader can get from participating in the SR market.

Although there is an infinite set of optimal dispatches with different associated levels of profit, there is only one Nash equilibrium in terms of bids. If either of the two leaders slightly increased its bid for SR, it would be too expensive and no spinning reserve would be awarded. On the other hand, if either of the leaders decreased its SR bid, it would be cheaper than the other leader and the competitive GenCo 14, and it would attract all 58.5 MW of SR. However, the profit that this leader would be making would be lower than if it could provide all the SR requirement at \$4.5/MWh, which still is one of the feasible dispatches that the ISO can choose.

7 Conclusions

Most models to analyze competition in the power sector are based on a lossless transmission network approach. Improved models of the transmission system are required in order to achieve more accurate outcomes. A novel formulation using standard NLP theory is presented to tackle two kind of games: the Cournot and the ASFE. A DC representation of the transmission system with a quadratic approximation of losses has been included in the formulation of the games. An illustrative example is used to compare the outcomes from the two kind of games. It is found that regardless of the rationality of competition, similar outcomes and effects are obtained when market power is exercised. It is also shown how strategic generators can exploit opportunity costs within integrated markets to arbitrage from one sub-market to another. A case of multiple optimal dispatches with a sole Nash equilibrium is discussed.

References

[1] Dennis W. Carlton and Jeffrey M. Perloff. *Modern Industrial Organization*. Foresman and company, 1990.

[2] J. F. Wilson. Scarcity, market power, and price caps in wholesale electric power markets. *The Electricity Journal*, 13:23–31, Nov. 2000.

[3] A. Y. Sheffrin, J. Chen, and B. F. Hobbs. Watching watts to prevent abuse of power. *Power and Energy Magazine, IEEE*, 2(4):58–65, July-August 2004.

[4] W. W. Hogan. A market power model with strategic interaction in electricity networks. *The Energy Journal*, 18(4):107–141, 1997. IAEE.

[5] C. J. Day, B. F. Hobbs, and J. S. Pang. Oligopolistic competition in power networks: A conjectured supply function approach. *IEEE Transactions on Power Systems*, 17(3):597–607, August 2002.

[6] G. Bautista, V. H. Quintana, and J. A. Aguado. An oligopolistic model for an integrated market of energy and spinning reserve. *IEEE Transaction on Power Systems*, 21(1):132–142, February 2006.

[7] G. Bautista, M. F. Anjos, and A. Vannelli. Formulation of Oligopolistic competition in AC networks: An NLP approach. *IEEE Transaction on Power Systems*, 22(1):105–115, February 2007.

[8] W. J. Yuan and Y. Smeers. Spatial oligopolistic electricity models with Cournot generators and regulated transmission prices. *Operations Research*, 47(1):102–112, Jan.-Feb. 1999.

[9] Scott M. Harvey and William W. Hogan. Market power and market simulations. Mimeo, Harvard University, [Online] Available at: http://ksghome.harvard.edu/ .whogan.cbg.Ksg/, July 2002.

[10] S. de la Torre, A. J. Conejo, and J. Contreras. Simulating oligopolistic pool-based electricity markets: A multiperiod approach. *IEEE Transactions on Power Systems*, 18(4):1547–1555, Nov. 2003.

[11] E. Hirst and B. Kirby, "Ancillary-service details: Operating reserves," ORNL/CON-452, Oak Ridge National Laboratory, Tech. Rep., November 1997.

[12] F. C. Schweppe, M. C. Caramanis, and R. D. Tabors. *Spot pricing of electricity.* Kluver Academic publishers, February 1988.

[13] S. Leyffer and T. Munson, "Solving multi-leader-follower games," *Preprint Argonne National Laboratory*, vol. ANL/MCS-P1243-0405, April 2005.

[14] R. Fletcher, S. Leyffer, D. Ralph, and S. Scholtes, "Local convergence of SQP

methods for mathematical programs with equilibrium constraints," Department of Mathematics, University of Dundee, NA/209, Tech. Rep., 2002.

[15] University of Washington, "Power systems test case archive," Sep. 2003, [Online]. Available at http://www.ee.washington.edu/research/pstca.

8 Tables

Table 1

Generation data for the 14-node system

i	α_i ($\$/MWh$)	β_i ($\$/MWh$)	γ_i ($\$/MW^2h$)	$\overline{g}_i$ (MW)
1	3	21	0.005	600
4	4	20	0.004	650
6	5	25	0.0075	600
9	3.5	22	0.006	500
14	4.5	22	0.005	500

Table 2

Demand data for the 14-node system

i	ρ_i ($\$/MWh$)	δ_i ($\$/MW^2h$)
3	70	5
5	70	5
6	65	5
8	65	5
10	65	5
11	70	5
12	60	5
13	75	5

Table 3

Cournot vs ASFE outcomes[a]

i	Competitive	Cournot	ASFE
g_1	358.6	345.3	345.3
g_9	500	324.1	327.9
g_4	470.6	524.8	523
g_6	600.0	600	600
g_{14}	253.3	254.4	255
r_1	196.5	58.7	0–58.5
r_9	0.0	1.3	0–58.5
r_4	0.0	125.2	127
r_6	0.0	0.0	0.0
r_{14}	0.0	0.0	0–58.5
$\Pi_1(g_1)$	642.9	4453.5	4440
$\Pi_9(g_9)$	2223	2588.3	2589.3
$\Pi_1(r_1)$	0.0	88	0–87.7
$\Pi_9(r_9)$	0.0	1.4	58.5
Π_1	642.9	4541.5	4440–4527.7
Π_9	2223	2589.7	2589.3–2647.8
Losses	217	196.3	196.7

[a] Generation, SR and losses in MW, and profits in \$/hr.

Table 4

Cournot vs ASFE prices and demands [b]

	Competitive		**C**ournot		**A**SFE	
	Price	Demand	Price	Demand	Price	Demand
1	24.6	–	35.6	–	35.6	–
2	30.0	–	36.9	–	36.8	–
3	34.1	356	37.8	317.7	37.8	318
4	23.8	–	24.7	–	24.7	–
5	47.4	223.1	49.3	202.5	49.2	203.2
6	40.3	243.7	40.3	242.4	40.3	242.4
7	27.5	–	29.4	–	29.4	–
8	27.5	372.1	29.4	351.1	29.4	351.6
9	29.4	–	31.9	–	31.9	–
10	39.9	247.8	41.9	226.6	41.8	227.2
11	50.3	193.5	51.2	183.8	51.1	184
12	51.7	79.8	51.5	80.0	51.6	80.1
13	49.8	249.4	49.7	248.1	49.7	248.2
14	24.5	–	24.5	–	24.6	–
SR	3	196.5	4.5	185.2	4.5	185.5

[b] Prices in $\$/MWh$, demand and SR in MW.