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Modeling Market Power in Electricity Markets: Is the Devil Only in the Details?

Basic approximations of the transmission system are ubiquitous in the literature on modeling competition in electricity markets. Because the main concern of market power is with active power, reactive power and voltage-related issues are commonly neglected, even though they are inherent features of an electrical power system. However, the usefulness of stylized formulations that do not comprise these system elements may be severely limited.

Guillermo Bautista, Miguel F. Anjos and Anthony Vannelli

Market power is one of the main concerns in today's electricity markets. Numerous mathematical models have been developed to study competition in electricity markets. As the transmission system has strong implications for market outcomes, the study of strategic behavior and market power assessment must account for its impact. Early models of strategic behavior either neglected the transmission system or incorporated it using its simplest approximation as a transportation network model. Unfortunately, this latter approach does not capture properly the features of electrical networks. The power flows in a transmission system are governed by physical laws, well-known as the current and voltage Kirschoff's laws. The path of power flows depends on the characteristics of the transmission system as a whole, and a particular path cannot be imposed. Given a

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transaction between two nodes, the power flows along different paths, giving rise to loop flows and impacting other transactions. Also, opposite flows in a transmission line result in only the net power flow taking place. Most other formulations use direct current (DC) linear approximations of the transmission system, and although this accounts for both current and voltage laws, reactive power and voltage constraints are neglected, even though they are inherent features of an electrical power system. A few models incorporate detailed representations of the transmission system by using an alternate current (AC) model.

The use of DC approximations is ubiquitous in the literature on modeling strategic behavior. However, the impacts of using such a simplification and the resulting bias in the outcomes are commonly overlooked. The inclusion of an AC system introduces more technical complications, though. We do not advocate that the AC model should be used in all the mathematical formulations of strategic behavior in electricity markets. Instead, we point out that a DC model may not be sufficient to capture the main features of any transmission system in computational simulations. Because the operation of power systems is a complex engineering problem, even pricing schemes in electricity markets require some degree of stylization. But if such stylizations do not capture key elements that have a direct impact on market outcomes, the usefulness of the resulting models for the assessment of market power may be severely limited. This article discusses some salient features of power systems that motivate the need for AC representations in modeling strategic behavior, explains why the DC models may fail to characterize such features, and uses simple examples to illustrate the difference in outcomes between formulations with AC and DC models.

The production of generation units is constrained in both active and reactive power.

I. Elements of a Power System

Most AC power system components have resistors, inductors, and capacitors. When a current passes through a resistor, power is transmitted from a source, say a generator, to the resistor. The amount of power transmitted fluctuates around a positive average value. This average value is the real or active power, measured in megawatts. In contrast, when a current passes through either an inductor or capacitor, half of the time the power is transmitted from the source to the inductor/capacitor and the rest of the time the power flows from the inductor/capacitor to the source. Hence, the amount of power transmitted fluctuates around an average value of zero, resulting in no delivery of useful power. This is reactive power, measured in mega volt-amperes reactive (MVAR). Although both are power, they are not added arithmetically, but rather geometrically, to compose apparent power measured in mega volt-amperes (MVA).

A. Generators

When modeling generators in a DC approach, the common practice is to account for maximum generation limits of active power. However, the production of generation units is constrained in both active and reactive power. A generator can provide different kinds of interrelated goods, such as active power, reactive power and spinning reserves. The capacity of a generator is limited by the generators' capability curve, well-known as the D-curve, which is defined by heating considerations. Such a curve sets a tradeoff between the production of active and reactive power.[1] The capacity of a generator to absorb or produce reactive power may be limited by the generator's current level of active power production. When a generator is operating on (or close to) any of the limits of its D-curve, and it is called upon to provide reactive power, it may have to reduce its provision of active power, and therefore forego profits from selling active power. With a DC approximation, the incentives imposed by the tradeoff between active and reactive power that a generator faces are not taken into account explicitly. Although the core of an electricity market is active power, and the cost of providing reactive power is low in comparison to the value of the active-power market, the importance of reactive power lies not only in the security of the system, but also in the generators' incentives and in the wholesale market performance.

B. Loads

Apart from the demand for active power, loads usually have a component of reactive power. For example, the reactive component can come from motors or fluorescent lighting. Like active power, reactive power may also be supplied by generators through the transmission system. However, the transmission of reactive power incurs higher voltage drops and losses than those produced by a comparable amount of active power. Transmission losses are proportional to the impedance of the conductor and the square of the current passing through it. Even in efficient transmission systems, losses are inevitable. Active power can be effectively transmitted over long distances. Reactive power, in contrast, has a local nature. An alternative solution (and operationally the most efficient) is to locally provide reactive power by other means, such as capacitor banks, synchronous condensers, or static VAR compensators. But at present, even with those alternatives, generators are called upon to provide reactive power support to some extent.[2] On the one hand, the fact that reactive power should be provided locally implies small (or even negligible) reactive loads and this somewhat validates the use of the DC models. On the other hand, in order to do so, some generators may modify the available capacity of active power.[3] To account for this in a DC model, some adjustments may be needed. However, making these adjustments may not be straightforward.

The importance of reactive power lies not only in the security of the system, but also in the generators' incentives and in the wholesale market performance.

C. Transmission lines

Transmission lines are used to carry electric power from one point to another. Unlike distribution lines, transmission lines are operated at high voltage levels in order to enable the efficient transmission of large quantities of power over long distances. A transmission line is commonly characterized by three parameters: (1) a resistance which is defined by the inherent resistivity of its conductors; (2) an inductive reactance that arises from the effect of the magnetic field in conductors; and (3) a shunt susceptance that arises from the capacitance effect of conductors created by an electric field. These parameters mainly depend on the length of the transmission line and on its geometry. As resistances are small compared to inductive reactances, resistances have a minor impact on power flows compared to the impact of inductive reactances. The shunt susceptance is usually neglected for short transmission lines, but the longer the transmission line is, the more significant its effect on the voltage profile.

The limiting factor of a transmission line is fundamentally physical; a capacity limit is imposed by thermal conditions. The current flowing through a transmission line produces losses and heat which impact on the operating temperature of the line. In order to avoid irreversible damage of transmission lines, temperature limits are imposed by means of a limit on the current that a line can transmit. In the operation of a transmission system, however, security limits are also included and they are usually defined as

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capacity limits on transmission lines, too. These limits arise from more involved technical issues in the system operation.[4]

II. DC Approximations and Their Limitations

A transmission network under steady state can be accurately modeled with AC power flows that relate four node variables: active power (generation/demand), reactive power (generation/demand), voltage magnitude,[5] and voltage angle. For a given set of network parameters, and knowing all voltage magnitudes and angles, active and reactive power flows on the transmission lines can be determined. As the accurate description of power flows requires complex expressions, their solution may be troublesome to obtain.[6] As a consequence, different simplifications of the electrical network have been developed.

A. Lossless approximations

A typical simplification of AC networks is the linearization of the power flow expressions. This linearization relies on three key assumptions[7]:

1. The resistance of transmission lines is small compared to their inductive reactance, and the shunt admittance is negligible. Thus, both can be disregarded, and then a transmission line is represented by its inductive reactance only.

2. Voltage angles, as well as differences in voltage angles, are assumed to be reasonably small. Hence, the sine function of voltage angle differences, which defines the power flows in a transmission system, can be approximated by the differences themselves.

3. There is sufficient reactive power compensation at all nodes to keep voltage levels constant at desirable levels, say, at one per unit value. So the voltage magnitude variables are disregarded from the representation.

A typical simplification of AC networks is the linearization of the power flow expressions.

With these assumptions, expressions and terms related to the reactive power flow are dropped, and the active power flow expressions are reduced to a set of linear relationships in terms of voltage angles. Due to these assumptions, the linear approximation is suitable to calculate active power flows and voltage angles, but it does not provide any insight regarding voltage magnitudes and reactive power flows.[8] This approximation also relies on the fact that there exists a strong coupling between active power and voltage angle, and reactive power and voltage magnitude. This feature is what validates, to some extent, the analysis of active power without explicit consideration of reactive power. Interestingly, there is no decoupling between the voltage magnitudes and voltage angles themselves; the voltage magnitude and angle at each node depend on the condition of the system as a whole.

The assumptions of the linear approximation give rise to a lossless power flow model, and the AC transmission system is represented simply as a passive network of resistors. One of the main advantages of a DC approximation is linearity, which allows the use of superposition. For instance, by using DC distribution factors, one can decompose and track individually the impact of each generator and each demand on the power flows. The distribution factors are constant and computed once for different system components, such as transmission lines and transformers.

B. Transmission line limits

In the study of strategic behavior in electricity markets, and particularly of issues related to congestion, it is common practice to account for the impact of the transmission system by using fixed thermal limits for the transmission lines only in terms of active power. The DC model only involves limits on active power flows, under the assumption that other system constraints can be approximated by equivalent line flow limits in terms of active power. In the case of transmission line constraints, the capacity limits are in apparent power (MVA) because the line has limits on both the current to carry and the voltage to operate. The combined effect imposes a limit on the amount of MWA to be transmitted, and therefore involves a reactive power component. Active and reactive powers consume transmission capacity and congest transmission lines to the same extent. To maximize the active power that can be transmitted, the reactive power flows must be minimized. The maximum active power that can be transmitted is achieved when there is no reactive power flow, i.e., a unity power factor (the power factor is the ratio of active power to apparent power). Because the DC model does not include either losses, voltage magnitudes, or reactive power, setting the transmission line limits with a unity power factor would possibly lead to higher active power flows than would result in the AC model. Apparent-related limits may be scaled down to get a corresponding active-only limit to be used with DC approximations. However, constraint limits have to be set to reasonable levels in consideration of the operating conditions of the system.

Transmission limits have to be enforced to guarantee a secure operation. Such limits are not constant, as they depend on the system condition.[9] System operators have to set the constraint limits to accommodate changing conditions imposed by the inherent nature of the system. With their comprehensive knowledge of the system status, only operators can arrive to such reasonable levels for constraint limits. When reactive power flows and voltage deviations are not negligible, the outcome of the DC model is not accurate and details neglected by the use of a DC approximation may become of paramount importance. Hence, the critical issue for modeling strategic behavior using DC models is to find the right value for capacity line limits which properly resemble the AC system conditions. This task is not trivial at all.

> *Voltage levels deviating from unity are a common occurrence in the operation of power systems.*

C. Voltage limits

A second major transmission constraint, and hence a source of congestion, is voltage constraints. The operation of a power system requires enforcing lower and upper voltage limits close to a nominal value of unity. This is because the power system elements (including all consumer equipment) are designed to operate within a nominal range of voltage. Operating outside of the nominal range may cause poor performance, shorten the equipment's operating life or damage the equipment.[10] The nominal voltage range also provides some degree of flexibility in the operation of the system to accommodate insufficient local provision of reactive power. Therefore, voltage levels deviating from unity are a common occurrence in the operation of power systems. Due to the enforcement of voltage constraints, the system can be congested even with no line thermal limits binding. As the flow of reactive power is always related to voltage drops, voltage constraints inherently limit the amount of reactive power that can be transmitted. Due to the strong coupling between voltage magnitude and reactive power, the voltage magnitude is mainly controlled by managing reactive power. The voltage magnitude at any node can be raised by injecting reactive power, whereas consumption of reactive power will lower voltage levels. Nonetheless, the system's requirements of reactive power depend on the transmission system configuration as well as on the location and levels of generation and demand. For instance, under low demand

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conditions, the capacitance of transmission lines may generate reactive power in excess that has to be absorbed, while under high demand, the system consumes reactive power that needs to be provided. This may happen even in the absence of reactive power demands.

Although a DC approximation may be adequate for characterizing congestion arising from the thermal limits of transmission lines, it may fail to characterize congestion arising from voltage constraints. In order to include voltage limits in the DC approach, certain transmission line limits can be scaled down accordingly. In a mesh network, however, the set of line limits to be scaled and the extent of the scaling are far from trivial. Only operators may have all the system information and the means to obtain a close estimate of the scaling, and even then this may be only valid for a specific operating point. Since the operating point continually varies, an estimate based on a particular operating point will likely be of little use in the modeling of competition in a general context with the DC model. When local voltage support is insufficient, the DC approximation fails to characterize the transmission system. In addition, prices generated by power flows may not characterize well the nature of voltage constraints. Moreover, locational marginal prices for active power are defined by all the system conditions, including reactive power flows and voltage limits.[11]

In some studies, such as the computation of available transfer capacity, DC approximations are used to quickly obtain an approximate solution and then the AC models are used to recover accuracy and explore further issues such as voltage-related constraints.[12] So a combination of the DC and AC approaches is sometimes needed within the same analysis to obtain the best tradeoff between accuracy in the modeling and simplicity in the implementation.

III. Illustrative Examples

We use a specific example to illustrate the most important differences in the modeling between the AC and DC approaches with the same setting of the transmission system, and using the same rationality of competition. The discussion is limited to the issues arising from transmission line flow limits and voltage constraints.

A. Model of the market and formulation of the competition

The study is based on a pool-like market in which a central entity receives voluntary bids from market participants, and simultaneously clears the market and manages the congestion via a cost minimization process, typically an optimal power flow. The outcome satisfies all the transmission constraints, balances the system at minimum cost, and provides locational marginal (or nodal) prices. Nodal prices reflect the locational value of power, which depends not only on the generation cost, but also on the transmission system characteristics and the demand's willingness to pay. Nodal prices may have generation, congestion, and losses components. In a lossless transmission system, even with a single binding transmission constraint, the nodal prices can be different. When losses are included, even with no transmission constraints binding, there will be different nodal prices.

To model competition, it is assumed that the generation companies are competing against each other in a Cournot fashion, while the market operator and the demands behave competitively as followers. This gives rise to the common approach of leader-follower games.[13] Such game formulations lead to hard nonconvex optimization problems, even if linear models of the transmission system are used. Moreover, due to the high nonconvexity, there is no guarantee of existence (or uniqueness) of equilibria. Indeed, such difficulties can arise even in a two-node system with a single transmission line using a DC approximation.[14] For the DC approach, different variants in the modeling have been proposed to make the game formulation more tractable. However, the differences in the modeling can lead to significantly different conclusions in the assessment of market power.[15]

In the following examples, reactive power is considered as a support for the system and it is not viewed as a commodity. Hence, profits are derived only from the sale of active power. In this way, reactive power is considered as a support and thus a means to enhance the exercise of market power in the active power market.

Let us consider the power system with three nodes shown in **Figure 1**. Two generation companies are modeled, one placed at node 1 (G1) and another placed at node 2 (G2). Generation costs are represented by convex quadratic functions. Both companies can provide active and reactive power, and are limited by linear piecewise D-curves. For both generators, we assume that at zero active-power generation, they can produce or absorb 600 MVAR, while at maximum active-power generation of 3,000 MW, they can produce or absorb up to 400 MVAR. A line connecting both points sets the limit of production for any other combination of active and reactive power. Also, there is a demand for both active and reactive power at each node. While demands for active power are given by affine functions, the inelastic demand for reactive power is 175 MVAR. For expository purposes, all transmission lines are assumed to have the same electrical characteristics,[16] and only the capacity limit of 350 MVA for the transmission line between nodes 1 and 3 is involved. In addition, upper (1.03 pu) and lower (0.97 pu) voltage limits for the three nodes are included.

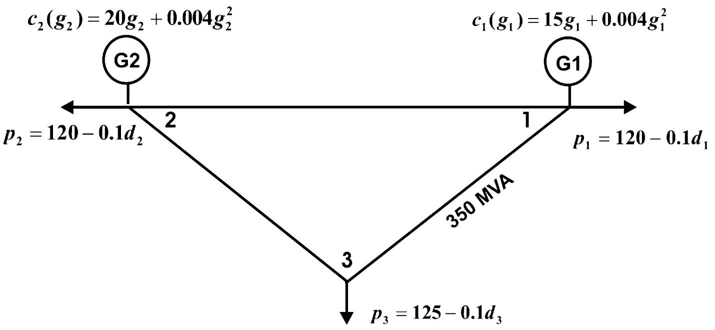

Figure 1: Three-Node System Used as an Example

B. Line and voltage constraints

First, a detailed AC system is used to model duopoly competition. The resulting generation schedules and power flows for both active and reactive powers are depicted in **Figure 2**. The first value of each pair stands for active power in MW and the second value for reactive power in MVAR. Also, voltage levels and active-power prices are given for each node. In this outcome, there are two transmission constraints on the limit: (1) the upper voltage at node 1; and (2) the capacity limit in line 1-3. Although they are on the limit, their corresponding congestion multipliers (dual variables) are both zero. This degeneracy is due to the fact that Cournot generators identify their impact on the transmission constraints, and therefore are able to weakly congest the system in order to capture all the system

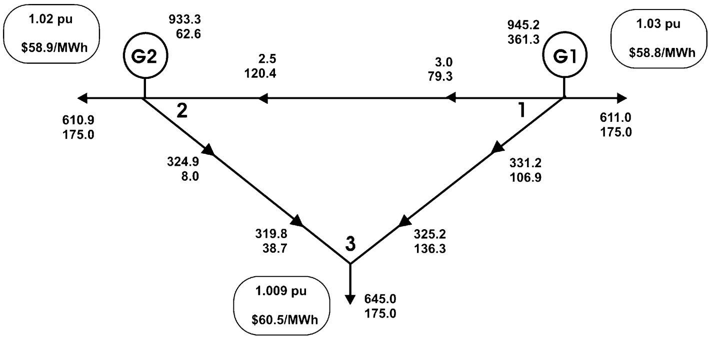

Figure 2: Market Outcome With an AC Model

congestion rents.[17] In an AC formulation, this feature holds not only for capacity line constraints but also for voltage constraints. The upper voltage limit is reached at node 1 because G1 is providing most of the reactive power, which tends to increase the voltage magnitude at its node. So the upper voltage limit is what implicitly bounds the level of reactive power generation from G1. The lowest voltage is at node 3, where the demand makes the voltage drop as there is no local support of reactive power.

The active power entering a transmission line is always higher than that coming out of it. The difference accounts for the transmission losses. For instance, in line 1-3 losses are 331.15–325.2 = 5.95 MW. The greater the amount of power transmitted, the greater the losses. On the other hand, the reactive power flows from high-voltage nodes to low-voltage nodes, and the power entering a transmission line is lower that that coming out. This apparently counter-intuitive effect is due to the line capacitance. However, reactive power flows can have similar patterns to that of active power, or even more complex patterns.[18]

Because of losses, it is common to deal with average power flowing through transmission lines. Therefore, capacity line limits are modeled as maximum average power in MVA. As reactive power is transmitted through the system, it requires transmission capacity and contributes to congestion to the same extent as active power. The power flow in line 1-3 is obtained by geometrically adding the averages of active and reactive power flows in this line, i.e.,

$$\sqrt{328.2^2\,\text{MW} + 121.6^2\,\text{MVAR}} = 350\,\text{MVA}.$$

Generators' profit is given by the difference of revenue and generation cost. The profit for G1 is

$$\begin{aligned} p_1 g_1 - c_1(g_1) &= 58.89 \times 945.18 \\ &\quad - (15 \times 945.18 \\ &\quad + 0.004 \times 945.18^2) \\ &= \$37,917.1/\text{hr} \end{aligned}$$

and similarly the profit for G2 is \$32,825.9/hr.

Second, using the same power system setting but with the DC approximation, voltage constraints and reactive demands are dropped, resistance and shunt susceptance of the transmission lines are neglected, and hence the capacity limit of the transmission line is defined as the active-only limit of 350 MW. The results are shown in **Figure 3**. Unlike for the AC model, the contribution of each generator to the power flows can be easily tracked. For instance, in order to satisfy the demand at node 3 with power from G1, two-thirds of the power flows through line 1-3 and one-third flows through lines 1-2 and 2-3.

Comparing the outcomes of the AC and DC models, we highlight the following features:

- In the DC approach, generators identify how they impact the system, and choose generation levels such that the line constraint is on the limit, while the corresponding congestion multiplier is zero and the system is weakly congested, as in the AC outcome. However, as no losses are considered, the weak congestion results in a unique market price of \$57.82/MWh. Although weak congestion also holds for the AC case, losses produce different prices. Hence, there is a qualitative difference in prices even though the numerical difference between them is less than 5 percent.
- There is a general mismatch of generation and demand levels. The most significant ones are the generation level of G1 which dif-

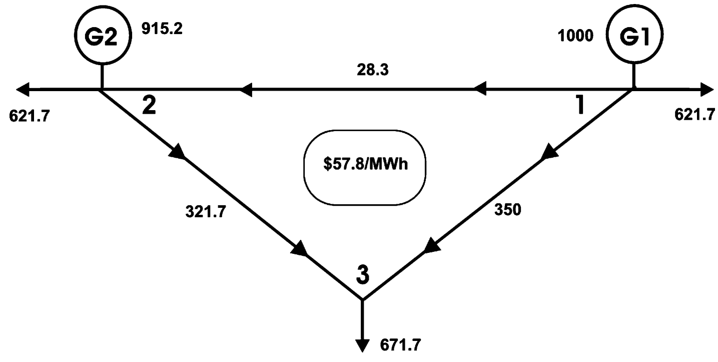

Figure 3: Market Outcome With a DC Approximation

fers by 1000–945.2 = 54.8 MW, and demand at node 3 which differs by 671.7–645 = 26.7 MW.

- Since the DC model is lossless, the amount of power entering any transmission line is the same as that coming out. Because reactive power is neglected, the capacity of line 1-3 is fully used by active power. This leads to higher generation from G1 (which is the cheapest unit). This results in marked variations on the power flows.
- Profits do not change so markedly. They change from \$37,917.1/hr (AC) to \$38,827.2/hr (DC) and from \$32,825.9 (AC) to \$31,268.7/hr (DC) for G1 and G2, respectively.

A natural possibility is to scale down the capacity limit of line 1-3 to a level for active-only power obtained from insights of the AC model. However, the issue is how to arrive to this level. Do we take a value based on a competitive outcome? If so, how good is an estimate from a competitive basis for use in a market power model? It is well known that Cournot competition may bias generation schedules away from the competitive point, and therefore power flows for both active and reactive under Cournot will vary, too. For instance, running a competitive scenario, the average active power flow in line 1–3 is 349.8 MW. If we use this limit for the simulation of market power with a DC approximation, the outcome will be quite close to the one of Figure 3, which uses a limit of 350 MW. So the use of the value from the competitive case is meaningless. Can we figure out a value of 328.2 MW (obtained from the simulation using the AC model) by other means? If so, that would imply that we can correctly estimate *ex ante* what the reactive power level would be with strategic behavior before even carrying out the market power assessment. But deciding the level of reactive power would be precisely the goal of modeling strategic behavior.

C. Voltage-only constraints

To show how these issues can be even more subtle, let us assume that there are no constraints on the transmission lines. In this case, only voltage constraints are enforced. The outcome for the AC model is given in **Figure 4**.

Generator G1 increases active power as it is cheaper; this makes G2 decrease active power generation. This adjustment also impacts on the levels of reactive power; now G2 provides all the reactive power, and G1 has to absorb an excess of it. The reason for this is that G2 increases reactive power in order to reach the upper voltage limit and exploit voltage constraints for profits. On the other hand, G2 has to decrease active power in order to increase the production of reactive power. As in previous cases, the voltage at one or more nodes (in this case, at node 2) is on the upper limit but its congestion multiplier is zero. Profits are \$43,128.9/hr and \$28,830.7/hr for G1 and G2, respectively, and losses increase to 14.9 MW. Although the voltage constraints do not induce congestion, different nodal prices arise because of the losses.

On the other hand, the outcome with a DC approximation is presented in **Figure 5**. Since no transmission constraints are in place, the outcome comes from a duopoly competition with only a power balance requirement, and hence there is a unique price in the system. Profits are now \$39,222.5/hr and \$30,510.9/hr for G1 and G2, respectively. It turns out that: (1) only one price is observed with the DC approach, and that price is lower than the nodal prices obtained with the AC model; (2) higher demand levels are observed at all nodes; and (3) the most vivid difference

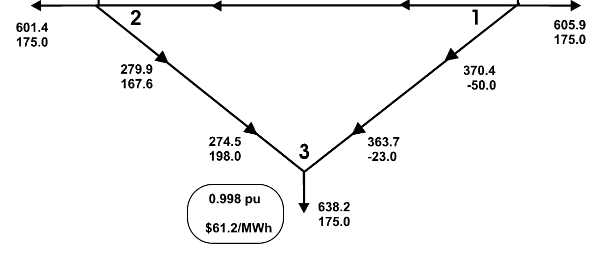

Figure 4: Market Outcome With No Line Limits

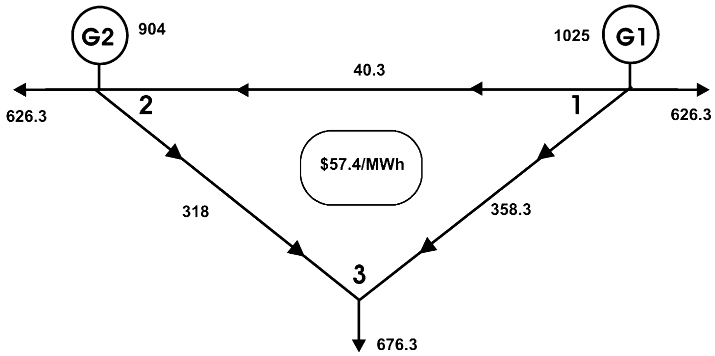

Figure 5: Market Outcome With No Line Limits and Using a DC Approximation

is in the generation levels. Indeed, G1 has a production of 1075.3–1025 = 50.3 MW less, while G2 increases generation by 904–785.1 = 118.9 MW. These marked differences arise from neglecting reactive power issues, as the active power production in the AC model is biased by the reactive power and voltage requirements. The fact that power systems are operated within a band of voltages is disregarded in the DC approximation.

The natural question for this case is then: What MW limits should we use for transmission lines to characterize this voltage-constrained market? The mismatch happens because G2 is constrained on the amount of active power it can produce by its current production of reactive power; this tradeoff is set by its capability curve. In the DC model, one may want to adjust the active power limit of generators to account for this fact. However, the use of the classical constant maximum generation limit requires knowledge of the reactive power levels that would take place under imperfect competition.

Accounting for the fact that generators have another decision variable in their production of reactive power captures the extra room they have in setting their strategy. The resulting strategies would simply not be captured using DC approximations. As an illustration, consider the outcome presented in Figure 6. It is another Nash equilibrium for the three-node system without line limits. We can see that this outcome is symmetric with respect to that given in Figure 4. Now G1 produces most of the reactive power, and G2 has to absorb an excess of it. Thus, voltage at node 1 is on the upper limit. Because of this, G1 reduces its active power, while G2 increases it up to 1,001 MW. Demand levels and prices are different, though, resulting in quite different profit levels. Profits are \$34,793.2/hr and \$37,544.2/hr for G1 and G2, respectively. This alternative equilibrium is a by-product of the reactive power and voltage-related issues, and it will not be captured with the DC model unless a different set of constraints is used.

IV. Final Remarks

When modeling competition in electricity markets, the challenging task is to recognize which features of the transmission system can be approximated, which ones need more accuracy, and which ones can be safely disregarded. In the context of economic policy, detailed representations may be unnecessary, and the data may be unavailable. The issue, however, is to what extent the use of simplified representations fails to characterize real-world markets which are operated under an AC basis, and may therefore provide misleading conclusions of market power. Nonetheless, there is a limitation to the degree of

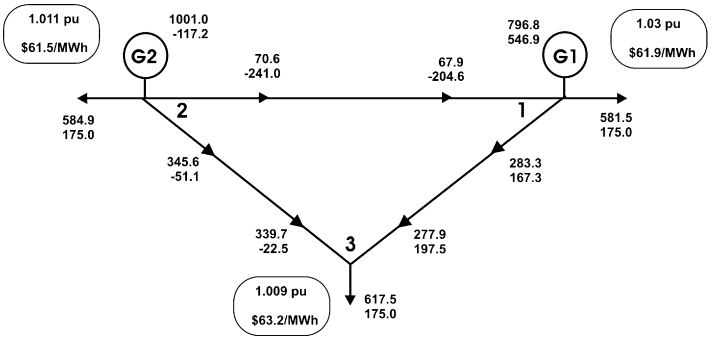

Figure 6: Another Nash Equilibrium with an AC Model

accuracy in the modeling that remains amenable to computation. Furthermore, higher accuracy in the modeling will require more data, which must be (publicly) available and reliable enough to warrant confidence in the results.

Finally, it is worth mentioning that the downside of having no losses in the DC approximation may be partially overcome by treating the losses as extra demands. One usual approach in competitive models is to place half of each line's losses at the nodes to which the line is connected. In some cases, the reported loads of a system are in fact composed by the true demand plus the estimated losses.[19] The task is then to estimate the level of losses to be used within the model of strategic behavior according to the expected outcomes. This area deserves research.■

Endnotes:

1. In the Pennsylvania-New Jersey-Maryland market, generators can provide between two and eight points so that the system operator builds a linear piecewise D-curve. Each point maps a value of active power to corresponding maximum generation and adsorption values for reactive power. See PJM Interconnection, *Reactive Power Review and Manual M14D Update*, Spring 2004.

2. Such sources, including generators, differ substantially in the speed and quantity of their provision of reactive power support, and also in their capital costs. Reactive power from generators should be used to fine tune the voltage profiles, while other sources, such as capacitors, should provide coarse reactive power to keep flat voltage around a unity power factor, as described in Ross Baldick, *Reactive Issues, Reactive Power in Restructured Markets*, IEEE POWER AND ENERGY, Dec. 2004, at 14–17.

3. Generators close to demands may be required to provide reactive power to meet local requirements, so the active power schedule could be limited by this practice, as described in Edward Kahn and Ross Baldick, *Reactive Power Is a Cheap Constraint*, ENERGY J., Vol. 15, Issue 4, 1994, at 191–201.

4. The technical issues are voltage drops, contingency, and stability constraints. The most common criterion for contingency is the (n-1) criterion. For stability limits, the constraint is related to the voltage angle differences, and this also depends on the length of the transmission lines.

5. Per-unit values are widely used in power systems to denote normalized figures in order to easily deal with different nominal levels of voltage in the system.

6. These power flow expressions are defined in terms of sines and cosines of voltage angle differences, and their products with voltage magnitudes. In addition, when solving AC power flows, the starting point may be hard to set, there can be low voltage solutions and convergence difficulties, and the process can be time consuming.

7. BEHIC R. GUNGOR, Power Systems (New York: Harcourt Brace Jovanovich, 1988), at 229.

8. ALLEN J. WOOD AND BRUCE F. WOLLENBERG, POWER GENERATION, OPERATION, AND CONTROL (New York: John Wiley & Sons, 1996), at 109.

9. Even ambient temperature and wind affect the capacity limits of transmission lines. For these reasons it is common to have different transmission limits depending on the season.

10. Brendan Kirby and Eric Hirst, *Ancillary Service Details: Voltage Control*, Report ORNL/CON-453, Oak Ridge National Laboratory, Dec. 1997, at 1.

11. William W. Hogan, *Markets in Real Electric Networks Require Reactive Prices*, Energy J., Vol. 14, Issue 3, 1993, at 171–199.

12. Ian Dobson, Scott Greene, Rajesh Rajaraman, Christopher L. DeMarco, Fernando L. Alvarado and Ray Zimmerman, *Electric Power Transfer Capability: Concepts, Applications, Sensitivity and Uncertainty*, PSERC Publication 01-34, Nov. 2001, at 17.

13. Details of the modeling and computational issues can be found in Guillermo Bautista, Miguel F. Anjos and Anthony Vannelli, *Formulation of Oligopolistic Competition in AC Power Networks: An NLP Approach*, IEEE TRANSACTIONS ON POWER SYSTEMS J., to appear in Feb. 2007.

14. Benjamin F. Hobbs and Udi Helman, *Complementarity-Based Equilibrium Modelling for Electric Power Markets*, in MODELLING PRICES IN COMPETITIVE ELECTRICITY MARKETS, Ed. Derek W. Bunn, (Hoboken, NJ: J. Wiley, 2004).

15. K. Neuhoff, J. Barquin, M.G. Boots, A. Ehrenmann, B.F. Hobbs and F.A.M. Rijkers, *Network-Constrained Models of Liberalized Electricity Markets: The Devil Is in the Details*, ENERGY ECONOMICS J., Vol. 23, Issue 3, 2005, at 495–525.

16. Resistance is 0.005 pu, reactive reactance is 0.01 pu, and shunt susceptance is 0.4 pu. These transmission parameters are taken from the reference given in *supra* note 11.

17. Shmuel S. Oren, *Economic Inefficiency of Passive Transmission Rights in Congested Electricity Systems with Competitive Generation*, ENERGY J., Vol. 18, Issue 1, 1997, at 63–83.

18. Depending on loading conditions, reactive flows can go on opposite directions in a transmission line. A line can either provide or consume reactive power at both ends at the same time. This can happen when voltage magnitudes at both ends are close in value.

19. Thomas J. Overbye, Xu Cheng and Yan Sun, *A Comparison of the AC and DC Power Flow Models for LMP Calculations*, Hawaii International Conference on System Sciences, Big Island, HI, Jan. 2004.