

$$M_+(K) := \left\{ Y \in \Sigma_+^{n+1} : \text{diag}(Y) = Y e_0; Y e_i, Y(e_0 - e_i) \in K \right. \\ \left. \forall i \right\}$$

$$N_+(K) := \left\{ Y e_0 : Y \in M_+(K) \right\}$$

$$N_+(P) := \left\{ x \in \mathbb{R}^n : \begin{pmatrix} 1 \\ x \end{pmatrix} \in N_+(K) \right\}$$

Then $K_I \subseteq N_+(K) \subseteq K$,
and $P_I \subseteq N_+(P) \subseteq P$.

If K is polyhedral (or SDP representable)
then optimizing a linear function over
 $M_+(K)$ is solving an SDP.

Defn. of dual cone: $K^* := \left\{ s \in \mathbb{R}^{n+1} : x^T s \geq 0 \right. \\ \left. \forall x \in K \right\}$

$$Q^* := \text{cone} \{ e_1, e_2, \dots, e_n, (e_0 - e_1), \dots, (e_0 - e_n) \}$$

$$M_+(K) = \left\{ Y \in \Sigma_+^{n+1} : \text{diag}(Y) = Y e_0, u^T Y v \geq 0, \forall u \in Q^*, v \in K^* \right\}$$