A Novel Eigenvector Technique for VLSI Layout

Dorothy Kucar
Laleh Behjat
Anthony Vannelli

University of Waterloo
Waterloo, Ontario

April 28^{th}, 2001

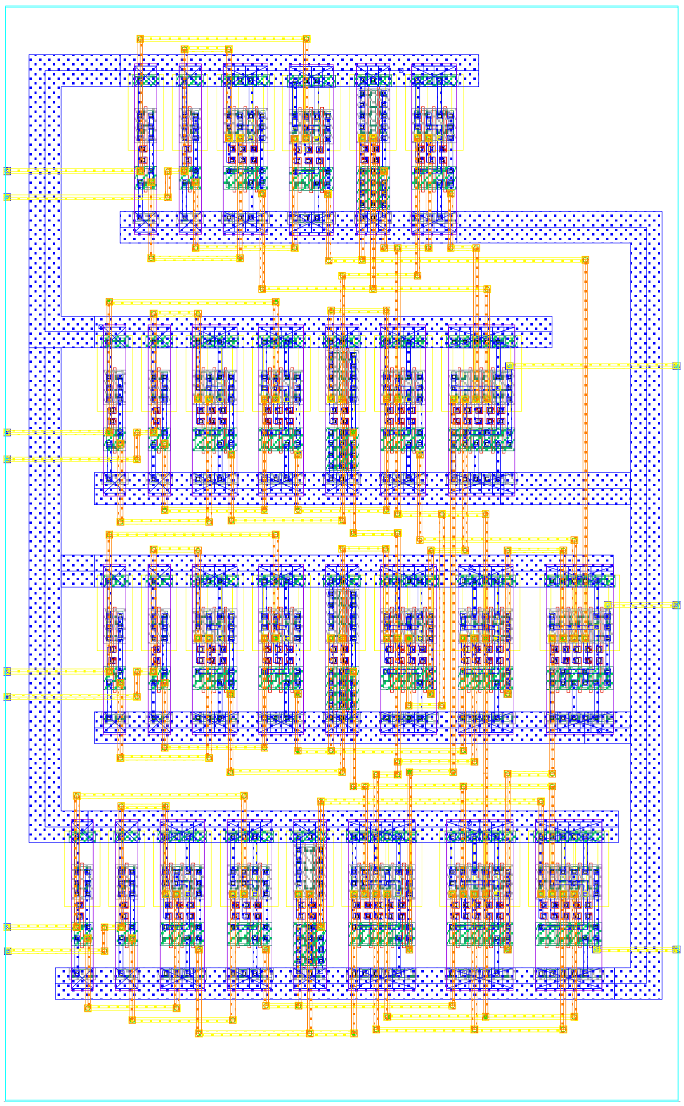
OUTLINE

1. Introduction
   (a) Goal
2. Eigenvector Method
3. Placement
   (a) Standard Cells
   (b) Placement Model
   (c) Placement Results
4. Routing
   (a) IP Formulation
   (b) Routing Model
   (c) Routing Results
5. Conclusion

1. INTRODUCTION

- today's integrated circuits contain millions of transistors which must be connected
- wirelength is the single most important factor in circuit performance criteria such as
  - signal delay
  - power dissipation
  - circuit size
  - co$t

- formulate circuit layout problem as a sequence of discrete optimization subproblems with the overall wirelength as the objective function
- these subproblems are:
  - *partitioning*: divide circuit into smaller, related subcircuits
  - *placement*: determine relative and absolute position of subcircuits within circuit
  - *routing*: determine course of wires
- circuit layout is an NP hard problem
- in particular, solving exact layout problem is an $O(n!)$ problem where n is number of subcircuits
- focus of past 15+ years of research has been on development heuristics to solve these problems approximately
- these techniques are not as effective as we would like



1A. GOAL

- Otten (1982) proposed a matrix reordering technique based on calculating one eigenvector to obtain an inexpensive global solution to the placement subproblem for uniformly–sized circuit elements
- in this paper, we adapt Otten's method
  - to the more general placement problem where each cell has uniform height but varying width
  - to facilitate matrix calculations in the discrete optimization formulation of the routing problem

2. EIGENVECTOR METHOD

- gist of Otten's method is to rearrange the rows and columns of a binary $n \times m$ matrix, $\mathbf{Q}$ such that the ones are clustered as tightly as possible along the diagonal

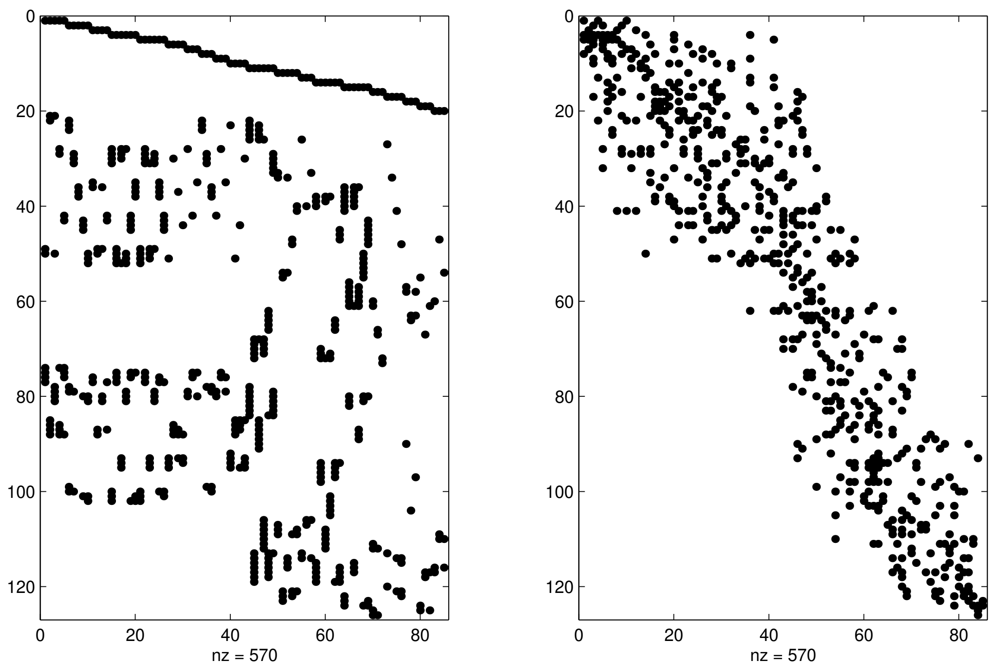


Figure 1: the effect of Otten's method

- given two vectors $\mathbf{r}$ and $\mathbf{k}$, both $\in \mathbf{R}^{n \times 1}$
- $\mathbf{r}^T\mathbf{k}$ is maximal when the components of $\mathbf{r}$ and $\mathbf{k}$ are simultaneously ordered from largest to smallest or vice versa
- extend reasoning to case where $\mathbf{r}$ and $\mathbf{k}$ are of different lengths by forming the matrix product $\mathbf{r}^T\mathbf{Q}\mathbf{k}$
- let coordinates of $\mathbf{r}$ and $\mathbf{k}$ represent row and column numbers of $\mathbf{Q}$; then $\mathbf{r}^T\mathbf{Q}\mathbf{k}$ is maximized when $\mathbf{r}$ and $\mathbf{k}$ are simultaneously ordered from largest to smallest and the non-zeros of $\mathbf{Q}$ are clustered along the diagonal

- Otten shows that given a “perfect” netlist:

$$\max \mathbf{r}^T \mathbf{Q} \mathbf{k} \quad = \quad \lambda$$

  and that

$$\begin{aligned} \mathbf{r} &= \mathbf{v} \\ \mathbf{k} &= \mathbf{C}_k^2 \mathbf{Q}^T \mathbf{v} \end{aligned}$$

  the components of $\mathbf{k}$ and $\mathbf{r}$ should be arranged from largest to smallest to give the correct column and row ordering

- the notation is defined by

$\mathbf{C}_k(j,j)$	$\dfrac{1}{\sqrt{\sum j\text{th column}}}$
$\mathbf{C}_r(i,i)$	$\dfrac{1}{\sqrt{\sum i\text{th row}}}$
$\mathbf{E}$	equals $\mathbf{C}_r \mathbf{Q} \mathbf{C}_k$
λ	2nd largest eigenvalue of $\mathbf{E}\mathbf{E}^T$
$\mathbf{v}$	corresponding eigenvector

3. PLACEMENT

- the simplest placement problem is given as

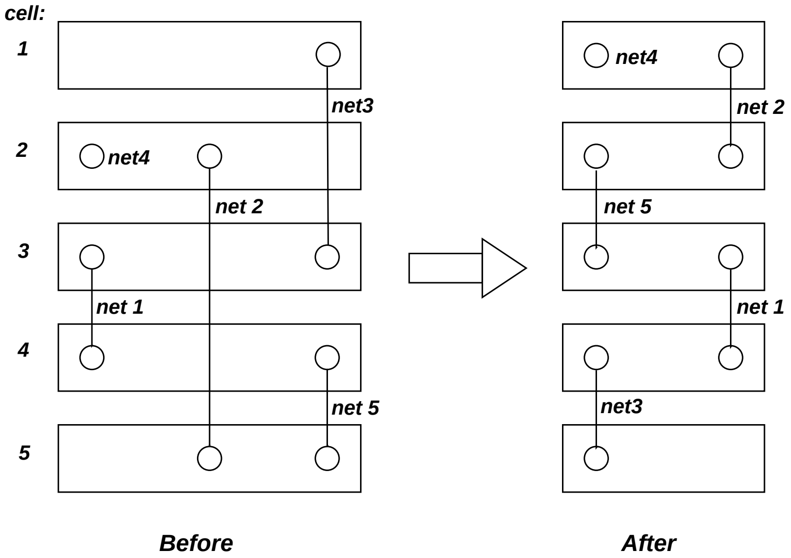
$$\min \quad \sum_{(r,s)} \sum_{l=1}^{m} \sum_{k=1}^{m} \left(|a_l - a_k| + |b_l - b_k|\right) x_{rk} x_{sl}$$

$$\text{s.t.} \quad \begin{cases} \sum_{j=1}^{m} x_{ij} = 1 & \forall\, i = 1, \ldots, n \\ \sum_{i=1}^{n} x_{ij} \leq 1 & \forall\, j = 1, \ldots, m \end{cases}$$

where

i	cell counter
j	location counter
k, l	location identifiers
n	number of cells: can be $O(10^7)$
$m \geq n$	number of locations
r, s	cell identifiers

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



3A. STANDARD–CELLS

- work with pre–designed functional circuit elements such as logic gates called *cells* with uniform height, pin structure but varying width

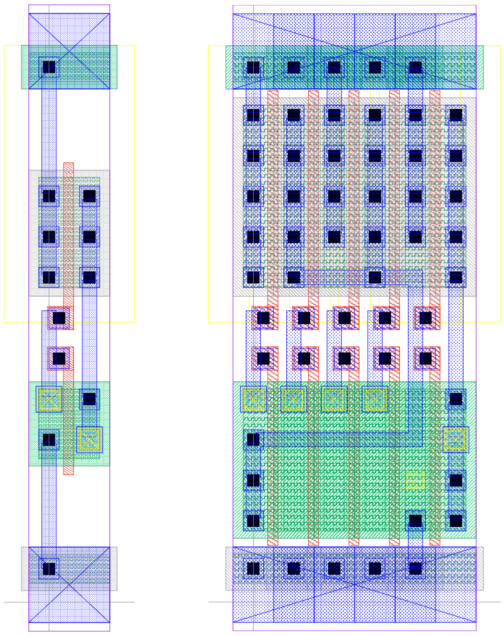


Figure 2: Inverter and 4–Input AND gate

3B. PLACEMENT MODEL

- since there are two degrees of freedom:
  - connections to other cells
  - size of cell
- we model this by introducing two connection matrices:
  - $\mathbf{Q}_{pin}$ which shows connections between cells
  - $\mathbf{Q}_{cell}$ which models size of cell

- let's say we have the following net description

cell	pin list	attached to pin
i2s	a, q	a
aoi22s	a_1, a_2, b_1, b_2, q	a_2
ai2s	a, b, q	q

- the 1st column of $\mathbf{Q}_{cell}$ would contain:

$$\begin{bmatrix} 2 & 2 & \dots & 5 & 5 & 5 & 5 & 5 & \dots & 3 & 3 & 3 \end{bmatrix}^T$$

- the first column of $\mathbf{Q}_{pin}$ would contain:

$$\begin{array}{c} \begin{array}{cccccccccccc} a & & & & a_2 & & & & & & & q \end{array} \\ \begin{bmatrix} 1 & 0 & \dots & 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{bmatrix}^T \end{array}$$

- we apply Otten's method to the $\mathbf{Q}_{cell}$ matrix
- we then reorder the rows and columns of $\mathbf{Q}_{pin}$ matrix according to the optimal row and column ordering obtained for the $\mathbf{Q}_{cell}$ matrix

3C. PLACEMENT RESULTS

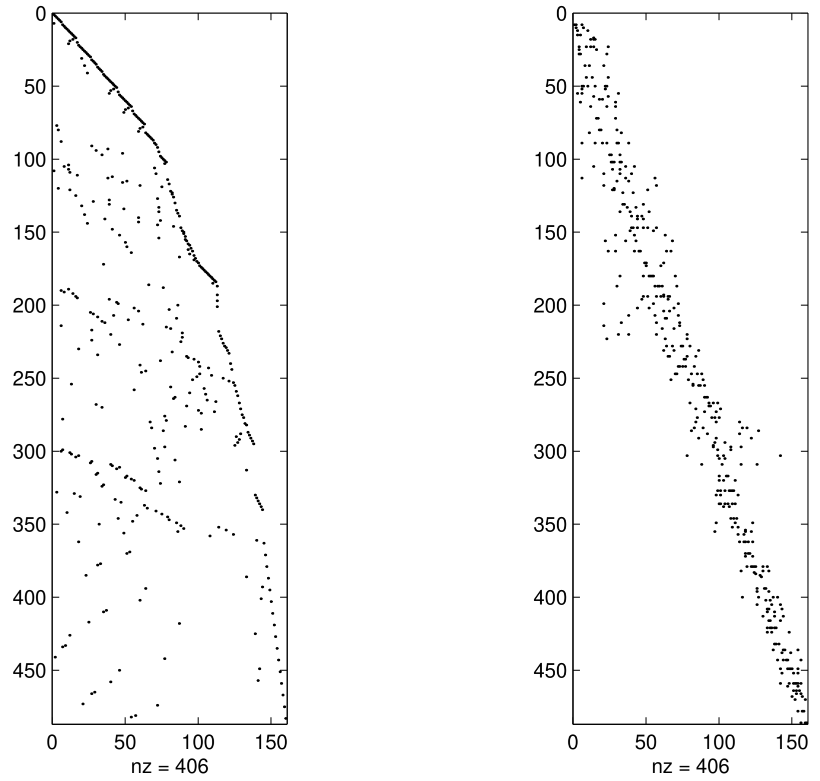


Figure 3: Initial and resulting pin arrangements

4. ROUTING

- the routing problem can be modelled as a linear programming problem

$$\begin{aligned} \text{min} \quad & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} \leq \mathbf{b} \end{aligned}$$

where

$$\begin{aligned} x_j \quad &= \begin{cases} \text{1 if tree j is used} \\ \text{0 otherwise} \end{cases} \\ \mathbf{A} \quad &= \text{ matrix of capacity and feasibility condition} \\ \mathbf{b} \quad & \text{ right hand side} \\ \mathbf{c} \quad & \text{ benefit of chosing tree } x_j \end{aligned}$$

4A. IP FORMULATION

- due to the large number of constraints, we solve this problem using an interior point method
- this involves finding the minimum of the corresponding Lagrangian equation:

$$L(\mathbf{r}) = f(\mathbf{x}) - \mu^k \sum_{i=1}^{m} (\ln s_i) - \boldsymbol{\pi}^T (\mathbf{A}\mathbf{x} + \mathbf{s} - \mathbf{b})$$

where $\boldsymbol{\pi}$ is a penalty parameter, μ is a barrier parameter and $\mathbf{s}$ are slack variables

- this can be solved approximately by taking a series of Newton steps which requires invering the reduced matrix:

$$\mathbf{A}^T \mathbf{D}_1^{-1} \mathbf{A} + \mathbf{D}_2$$

where $\mathbf{D}_1$ and $\mathbf{D}_2$ are diagonal matrices having nonnegative values

4B. Routing Model

- before the matrix inversion is performed, the matrix is reordered to pack the non–zero entries as close to the diagonal as possible
- this reduces the number of fill–ins in the LU factorization as compared to Min–Fill Factorization method

4c. Routing Results

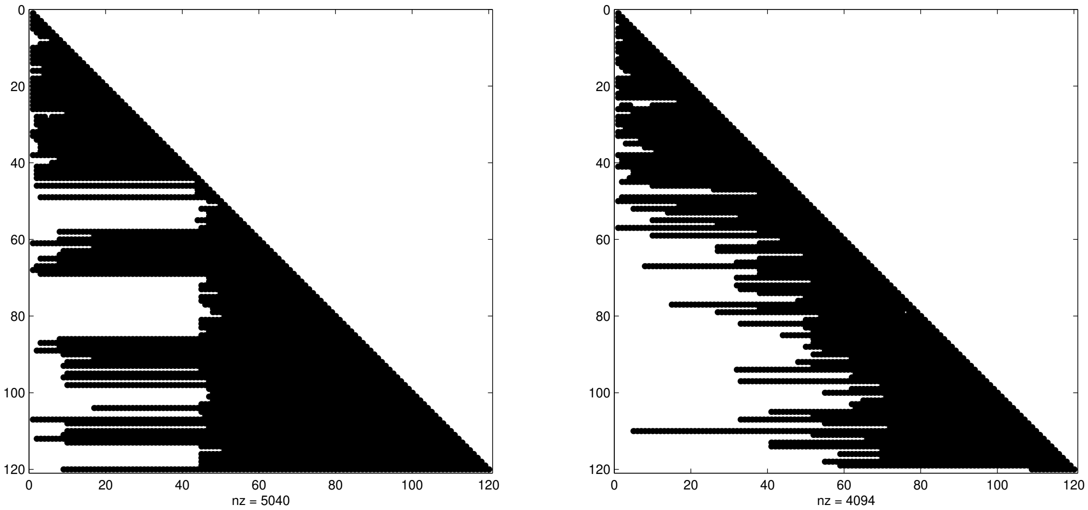


Figure 4: Sparsity pattern of the lower triangular matrix resulting from the Min-Fill Factorization (left) and the Otten Approach (right)

5. CONCLUSION

- we have shown the utility of a novel eigenvector technique to two types of problems arising in VLSI layout
- a good global placement was obtained in polynomial time where the polynomial is of low degree
- in the routing problem, the computational complexity of the Newton–step calculation was greatly reduced