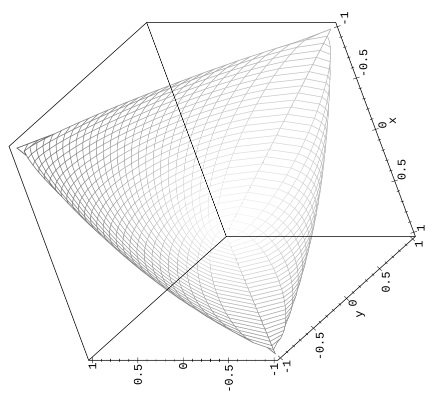
Workshop on
Semidefinite Programming and Interior-Point Approaches for Combinatorial Optimization Problems



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Semidefinite Programming (SDP) is a generalization of Linear Programming (LP) in that the nonnegativity constraints on the variables is replaced by a positive semidefinite constraint on matrix variables. Many of the elegant theoretical properties and powerful solution techniques follow through from LP to SDP. In particular, the primal-dual interior-point methods, which are currently so successful for LP, can be used to efficiently solve SDP problems.

In addition to the interesting theoretical and algorithmic questions, SDP has found many important applications in Combinatorial Optimization, Control Theory, Statistics, and other areas of Mathematical Programming. SDP is currently a very hot area of research. This can be seen by the number of talks on SDP at various recent optimization conferences and the number of recent publications, e.g. [4, 11, 1, 10].

The workshop on SDP at the Fields Institute attracted roughly 100 researchers. Participants from Australia, Austria, Brazil, Belgium, Canada, Israel, France, Hungary, Italy, The Netherlands, Puerto Rico, and USA gave the workshop an important international component. There were 39 talks during the three days of the workshop. (A list of participants, speakers, titles, abstracts, and this article, can be found with URL: http://orion.uwaterloo.ca/~hwolkowi/fields.d/readme.html) Following is a short, far from comprehensive, outline of several of the talks.

The workshop started with two talks on **Matrix Completion Problems**, i.e. under what circumstances a partial matrix has a completion of a desired type. These problems have been studied extensively beginning in the early 80's, and exemplifies one of the early instances of SDP, e.g. [7].
Monique Laurent (Ecole Normale Superieure) spoke on: "A Connection Between Positive Semidefinite and Euclidean Distance Matrix Completion Problems". Although there is a strong relationship between positive semidefinite matrices and Euclidean distance matrices, it was not clear how to link the two completion problems. Monique showed how the results for the Euclidean distance matrix completion problem can be derived from the corresponding results for the positive semidefinite completion problem, using a functional transform introduced by Schoenberg.
Charlie Johnson (College of William and Mary) spoke on "Recent Progress on Matrix Completion problems". After discussing the state of the art on positive definite completion problems, he continued with various other completion problems including those for totally positive matrices, P-matrices, inverse M-matrices, completely positive and doubly nonnegative matrices.

There were many talks on applications to **Combinatorial Optimization.** One of the main reasons for the current interest in SDP is the success in finding good approximations for the max-cut problem, e.g. [5, 6]. However, SDP has applications to many other combinatorial problems, e.g. [11, 1].

Stefan Karisch (University of Graz) spoke on: "Semidefinite Programming and Graph Equipartition". Stefan showed how SDP can be used to approximate the problem of partitioning a graph into equally sized components. Improvements were shown on previous eigenvalue approaches.

Christoph Helmberg (University of Graz) spoke on "Quadratic Knapsack Relaxations Using Cutting Planes and Semidefinite Programming". Though the quadratic knapsack problem is extremely difficult to solve by linear programming alone, it was shown that SDP is very useful for quadratic knapsack problems.

Hsueh-I Lu (Brown University) spoke on: "Approximation Algorithms for Semidefinite Programs arising from Max Cut and Coloring".

Tamas Terlaky (Delft University of Technology) spoke on: "Potential Reduction Algorithms for Structured Combinatorial Optimization Problems". Tamas presented a modified potential function, for binary feasibility problems, that is computationally more attractive than the existing ones. A special class of binary feasibility problems were reformulated as nonconvex quadratic optimization problems. The reformulation is very compact. Computational results on several instances of the graph coloring and frequency assignment problem were presented. These results compared three different potential functions.

Qing Zhao (University of Waterloo) spoke on: "Semidefinite Programming Relaxations for the Quadratic Assignment Problem". Qing used the special structure of QAP to construct a *gangster* operator which enabled him to work in the minimal face of the feasible set. By exploiting this special structure and also using a conjugate gradient method, he was able to get strong bounds for the Nugent test set.

Many talks concentrated on **Primal-Dual Interior-Point Methods** for SDP. One of the reasons for the success of SDP is that interior-point approaches from LP can be extended to SDP; though the extension is not completely straightforward, e.g. [8]. Interesting complications can arise such as duality gaps, [9], lack of strict complementary slackness, [2], and confusing choices in the complementarity equations, [3].

Kees Roos (Delft University of Technology) spoke on: "Initialization in Semidefinite Programming via a Self-Dual Embedding". In this way the initialization problem for semidefinite problems can be solved nicely. The method also provides a solution for the initialization of quadratic programs and it is applicable as well to more general convex problems with conic formulation.

Zhi-Quan Luo (McMaster University) spoke on "Superlinear Convergence of a Symmetric Primal-Dual Path Following Algorithm for Semidefinite Programming ".

Michael J. Todd (Cornell University) spoke on: "The Nesterov-Todd direction in semidefinite programming ". Mike showed how to compute the direction efficiently and how to view it a s a Newton direction.

Romesh Saigal (University of Michigan) spoke on: "An Infeasible Start Predictor Corrector Method for Semidefinite Linear Programming".

Shuzhong Zhang (Erasmus University Rotterdam) spoke on: "Symmetric Primal-Dual Path Following Algorithms for SDP".

Yin Zhang (University of Maryland) spoke on: "Infeasible Primal-Dual Interior-Point Methods for Semidefinite Programming". Yin presented formulations, or symmetrization schemes, for the complementarity condition to obtain square optimality systems so that Newton-type methods can be applied. He also discussed a complexity theorem for an infeasible, long-step, path-following algorithm and several computational issues with preliminary numerical results.

Several talks discussed the **Geometry, Duality, and Complexity** of solving SDP:

Franz Rendl (University of Graz) spoke on: "Large Scale SDP using Eigenvalues". The advantage of this approach lies in the fact, that extreme eigenvalues of symmetric matrices can be computed without having the matrix explicitly available.

Gabor Pataki (University of Waterloo) spoke on: "Cone-LP's and Semidefinite Programs: Geometry and a Simplex-type Method ".

Motakuri Ramana (University of Florida) spoke on: "Strong Duality for Semidefinite Programming". Unlike LP, a duality gap can occur in SDP. Two approaches for closing the gap are compared and shown to have a common basis, though one is of polynomial size while the other is not.

Jos F. Sturm (Erasmus University Rotterdam) spoke on: "Duality and Self-Duality for Semidefinite and Conic Convex Programming".

Lorant Porkolab (Rutgers University) spoke on: "Bounds on Feasible Solutions of Semidefinite Programs".

Leonid Faybusovich (Notre Dame University) spoke on: "Infinite-dimensional Semidefinite Programming: Self-Concordant Barriers and Path-Following Algorithms for Semidefinite Programming".

Alexander Shapiro (Georgia Institute of Technology) spoke on: "Second Order Optimality Conditions and Stability Analysis of Semidefinite Programs".

Katya Scheinberg (Columbia University) spoke on: "Interior Point Trajectories in Semidefinite Programming".

Manuel A. Nunez (MIT) spoke on: "Condition Measures and Properties of the Central Trajectory of a Semidefinite Program". Manuel used Renegar's condition number to provide bounds on solution size and rates of change of solution. This shows that Renegar's condition number can greatly simplify sensitivity analysis for SDP.

Other **applications of SDP** were introduced and discussed. The wide ranging applicability, as well as fascinating and intriguing theoretical qualities, shows that SDP promises to be an active discipline of mathematical programming for many years to come.

Lieven Vandenberghe (Stanford University) spoke on: "Determinant Maximization with Linear Matrix Inequality Constraints". This problem has many interesting applications including finding the ellipsoid of minimal volume that contains a given polytope or given points.

Arjan Berkelaar (Erasmus University Rotterdam) spoke on: "Convergence Issues and Path-following Algorithms for Semidefinite Programming ". Arjan discussed convergence issues related to recent primal-dual interiorpint algorithms for the monotone semidefinite linear complementarity problem.

Renato Monteiro (Georgia Institute of Technology) "On Two Interior-Point Mappings for Nonlinear Semidefinite Complementarity Problems". Renato presented properties of two fundamental mappings associated with the family of interior-point methods for solving monotone nonlinear complementarity problems over the cone of symmetric positive semidefinite matrices. The first of these maps lead to a family of new continuous trajectories which include the central trajectory as a special case. The trajectories completely "fill up" the set of interior feasible points of the problem in the same way as the weighted central paths do the interior of the feasible region of a linear program.

Boris Mirkin (DIMACS, Rutgers University) "A Mine of Semidefinite Programming Problems".

In addition, **Related Problems** presented included:

Ding-zhu Du (University of Minnesota) spoke on "On Floorplan Design and Optimization".

Jun Gu (Hong Kong Univ. of Science and Technology) spoke on "Parallel Algorithms for Satisfiability (SAT) Problem".

Linas Mockus (AT&T Bell Laboratories) spoke on "Bayesian Approach to Combinatorial Optimization".

Mauricio Resende (AT&T Bell Laboratories) spoke on "Using linear programming to help solve quadratic assignment problems". The linear programming problems used to obtain lower bounds for quadratic assignment problems are very large and highly degenerate. For such problems simplex type algorithms or interior point methods based on direct factorizations, can only handle small instances. Large instance can be successfully solved with an interior point algorithm that uses a preconditioned conjugate gradient approach to approximately compute interior point directions.

M. R. Emamy-K (University of Puerto Rico) "How efficient can we maximize threshold pseudo-Boolean functions ?" The class of threshold pseudo-boolean functions was introduced by P.L. Hammer et. al. about 10 years ago. Existence of a polynomial algorithm to maximize these functions has been an open problem since then.

Pham Dinh Tao (LMI-INSA Rouen) "D.c. (difference of convex functions) Optimization: Theory, Algorithms and Applications". A general discussion of d.c. optimization was presented. Its importance lies in the fact that d.c. programming marks the passage from convex optimization to nonconvex optimization.

Laura Palagi (Universit'a di Roma "La Sapienza") "Trust region Problems: Theoretic Results and New Algorithmic Developments ". The trust region subproblem is important in nonlinear programming as well as other applications in combinatorial optimization. The special structure of the problem allows one to solve an equivalent unconstrained minimization of a piecewise quartic merit function.

Le Thi Hoai An (LMI-INSA Rouen) "An efficient adapted DCA and Branch-and-Bound algorithm for globally solving large-scale 0-1 quadratic programming problems".

A proceedings of the workshop will be published by the American Mathematical Society in the FIELDS Series.

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