HYPERBOLIC SYSTEMS: assignment 2

Use Matlab, fortran, C, ... to do the following simulations.
You may start from the fortran90 codes shallow.f90 (for the 1D shallow water MHD system) or laxmpi.f90 (for the 2D scalar Burgers equation, parallel with mpi), but it may be the easiest to just program the (simple) schemes in matlab yourself.

1 simulation of the Burgers equation

-do one set of simulations with $u_l = 1$, $u_r = 2$ for the left and the right state in a Riemann problem, and do another set of simulations with u_l and u_r switched
-use the following schemes:

(a) the FOU scheme (generalized for a nonlinear equation) (this is the local Lax-Friedrichs scheme)

(b) the LW scheme, written in conservative form, and extended for nonlinear equations (formulate such an extension)

(c) the non-conservative scheme of p. 2-129 in the lecture notes

-check that the rarefaction rarefies, and the shock propagates, with the right speeds
-compare accuracy, stability (maximum timestep), positivity, conservation (right shock speed) for the various schemes
-is the rarefaction solution correct for the non-conservative scheme? is the shock solution correct? why the different behavior?

2 extension to second order scheme or system of equations

either (a) **or** (b)

(a) **second order upwind scheme**: implement the nonlinear second order scheme that uses a slope limiter, and repeat the simulations of question 1

do you observe improvement over the FOU scheme?

(b) 2×2 **system** from question 3a or 3b of assignment 1: simulate the Riemann problem with states U_1 and U_2 that you obtained from the Rankine-Hugoniot relations for the system 3a or 3b, using the nonlinear FOU scheme generalized for systems

do you obtain a shock or a rarefaction?

switch the left and the right states and repeat