Homework assignments

Guidelines

- choose any part of the four questions, for a total of at least 100 'points'
- if you want to do the programming parts, start from the 2D program that was discussed in the lectures, to be found on the SP2 at /home/pgaha12/homework
- if you want to do the parallel programming parts, start from the parallel 2D program that was discussed in the lectures, to be found on Alfven at /home/hans/mpi/homework (ask me for a login on Alfven)

1. MHD waves

- A (10 points)

  - prove that Alfven waves are linearly degenerate

  - prove that fast and slow waves are not linearly degenerate

- B (30 points)

  - prove that fast and slow waves are non-convex

- C (40 points) : programming

  - write a 1D simulation program starting from the program discussed in the lectures (use the first order Lax-Friedrichs scheme)

  - do the following simulations on a background with $\rho = 1, p = 1, B_x$=1, $v_x = 3$, $B_y = B_z = v_y = v_z = 0$

  (a) a right moving Alfven wave packet ($\sim R_{alfven}$)

Is the shape preserved? Why? Does changing the amplitude have any influence?

(b) a right moving fast wave packet ($\sim R_{fast}$, thus linearized)

Is the shape preserved? Why? Does changing the amplitude have any influence?

Do you observe shock formation? Rarefaction?

(c) simulate left going Alfven and fast wave packets

What happens if you change the sign of the background B_x?

- D (20 points, plus the 40 points of C): parallel programming

  - implement the program of C for parallel computing

  - do the simulations of C on a parallel computer

  - perform a parallel performance study (determine speed-up curve)

  - determine how the execution time scales as a function of the grid resolution, both experimentally and by analyzing the algorithm

2. MHD discontinuities

- A (50 points)

  - derive the conditions under which switch-on shocks arise from the MHD RH relations

  - are these also the conditions under which intermediate shocks can occur?

- B (50 points)

  - the 2D MHD wedge problem: given the upstream ρ, $\vec{v}$, $\vec{B}$, p, and the wedge angle θ, find ψ and the downstream state (implicit transcendent formula, graphical method, numerically: see mathematica program)

  - how many solutions can you find in general?

  (for Euler there are generally two solutions, look in the Courant and Friedrichs book (1944), Section 122)

3. 2x2 hyperbolic systems

- the Shallow Water Equations are given by

$$\frac{\partial}{\partial t}\begin{bmatrix} v \\ \phi \end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix} v^2/2+\phi \\ v\,\phi \end{bmatrix} = 0.$$

- the Isentropic Euler Equations are given by

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho\, v \end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix} \rho\, v \\ \rho\, v^2 + s\,\rho^\gamma \end{bmatrix} = 0, \text{ with } s \text{ and } \gamma \text{ constants.}$$

$\Rightarrow$ for either of these equation sets (or both, double the points then), do

- A (20 points)

  - is the system hyperbolic? give eigenvalues, and left and right eigenvectors

- B (20 points)

  - derive the Riemann Invariants of the two wave modes

- C (20 points)

- are the wave modes linearly degenerate?

- if not, are they convex or non-convex?

- C (20 points) : programming

  - write a 1D simulation program starting from the program discussed in the lectures (use the first order Lax-Friedrichs scheme)

  - do the following simulations (choose a background state)

  simulate the propagation of a wave packet for the two modes ($\sim R_i$, thus linearized)

  Is the shape preserved? Why? Does changing the amplitude have any influence?

  Do you observe shock formation? Rarefaction?

  - simulate a simple wave

- D (20 points, plus the 20 points of C): parallel programming

  - implement the program of C for parallel computing

- do the simulations of C on a parallel computer

- perform a parallel performance study (determine speed-up curve)

- determine how the execution time scales as a function of the grid resolution, both experimentally and by analyzing the algorithm

4. Dispersion for linear waves

- A (20 points)

  - what is dispersion in linear wave theory? (look up one or more definitions, and give the reference in your answer)

- B (20 points)

  - are linearized sound waves (Euler) dispersive in 1D? in 2D?

- C (40 points)

  - are linearized fast and slow MHD waves dispersive in 1D? in 2D?