CO 750 Randomized Algorithms (Winter 2011)
Assignment 1

Due: Thursday January 27th.

Question 1: Prove ZPP = RP $\cap$ coRP.

Question 2: Consider a sequence of n unbiased coin flips. Consider the length of the longest contiguous sequence of heads.

(a): Show that you are unlikely to see a sequence of length $c + \log_2 n$ for $c > 1$. Give a decreasing bound as a function of c.

(b): Show that with high probability you will see a sequence of length $\log_2 n - O(\log_2 \log_2 n)$.

Question 3: Given a point x in $\{0, 1\}^n$, its weight is

$$wt(x) := \Delta(x, 0) = \|x\|_1 = \sum_i x_i.$$

A code $C \subseteq \{0, 1\}^n$ is called a ***constant weight code*** if $wt(x) = wt(y)\ \forall x, y \in C$. Prove there exists a constant weight code of distance $1/2 - \epsilon$ and rate $\Omega(\epsilon^2)$ where every codeword has weight $n/2$.

Question 4: Let $X_1, ..., X_n$ be mutually independent random variables which have the geometric distribution with mean 2. Let $X = \sum_{i=1}^{n} X_i$ and let $\delta > 0$. Use a Chernoff-based approach to derive an upper bound on $\Pr[\, X \geq (1 + \delta)2n \,]$.