CO 355 Mathematical Optimization (Fall 2010)
Assignment 0 (Preliminary Quiz)

Due: Tuesday September 21st, in class.

Policy. No collaboration is allowed. You are welcome to seek help from the current instructor and TAs for CO 355.

Question 1: Why did you decide to take this class? Although it's perhaps hard to say, what are your future career plans (or a few possibilities)?

Question 2: True or false:

(a): To prove a logical statement, it is sufficient to prove its converse.

(b): To prove a logical statement, it is sufficient to prove its contrapositive.

Question 3: Write the negation of the statement "John ate a burger and Fred drank a beer".

Question 4: True or false:

(a): Let A be a matrix, let u and v be vectors, and let α and β be scalars. Is

$$A(\alpha u + \beta v) \;=\; \alpha A u + \beta A v?$$

(b): Let x and y be linearly dependent vectors. Is x is a scalar multiple of y?

(c): Let A and B be square matrices of the same size and such that $\det A = \det B$. Is $\operatorname{rank} A = \operatorname{rank} B$?

(d): Let A be a non-square matrix. Is $\operatorname{rank}(A) = \operatorname{rank}(A^{\mathsf{T}})$?

Question 5: Yes or no:

(a): Is the following matrix product well-defined?

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(b): Is the following determinant well-defined?

$$\det\left(\begin{bmatrix} 3 & 0 & 1 \\ 2 & 1 & 2 \end{bmatrix}\right)$$

(c): Is the following matrix product well-defined?

$$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

(d): Is the following determinant well-defined?

$$\det\left(\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}\right)$$

(e): Is $\begin{bmatrix} 1 & 3 \\ 7 & 9 \end{bmatrix}$ a submatrix of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$?

Question 6: Let A be an $m \times n$ matrix and let B be an $n \times p$ matrix. Prove that $(AB)^{\mathsf{T}} = B^{\mathsf{T}} A^{\mathsf{T}}$.

Question 7: Let A be a square, invertible matrix. Prove that $(A^{-1})^{\mathsf{T}} = (A^{\mathsf{T}})^{-1}$.

Question 8: Let A and B be $n \times n$ matrices. Suppose that $v \in \mathbb{R}^n$ is an eigenvector of A and also an eigenvector of B. Prove that v is also an eigenvector of $A + B$.

A set $S \subseteq \mathbb{R}^n$ is called ***convex*** if for every $x \in S$ and $y \in S$ and every scalar $\lambda \in [0, 1]$, it holds that $\lambda x + (1 - \lambda)y \in S$.

Question 9: For every $\alpha \in \mathbb{R}$, let $S_\alpha \subseteq \mathbb{R}^n$ be a convex set. Give a proof or a counterexample to the following questions.

(a): Let $N = \bigcap_{\alpha \in \mathbb{R}} S_\alpha$. Is N convex?

(b): Let $U = \bigcup_{\alpha \in \mathbb{R}} S_\alpha$. Is U convex?

Question 10: Let A be a matrix of size $m \times n$ and let b be a vector in $\mathbb{R}^m$. Define $S = \{\, x \in \mathbb{R}^n \,:\, Ax = b \,\}$. Prove that S is convex.