Lecture 9: CS Example

Monday, October 12, 2009
9:34 AM

$$\begin{array}{lrcl}
\min & 5x_1 + 6x_2 - x_3 & & \\
\text{s.t.} & x_1 + x_2 & = & 3 \\
& x_1 + 2x_2 + 3x_3 & \leq & 8 \\
& 4x_2 + 5x_3 & \geq & 2 \\
& x_2, x_3 & \geq & 0
\end{array}$$

Claim: $(3, 0, 5/3)$ is optimal. Obj val $= 15 - 5/3$

How to prove it?

Dual

$$\begin{array}{lrcl}
\max & 3y_1 + 8y_2 + 2y_3 & & \\
\text{s.t.} & y_1 + y_2 & & \\
& y_1 + 2y_2 + 4y_3 & \leq & 6 \\
& 3y_2 + 5y_3 & \leq & -1 \\
& y_2 \leq 0, & & y_3 \geq 0
\end{array}$$

CS conditions:

1. 2nd primal tight or $y_2 = 0$ (Already satisfied)
2. 3rd primal tight or $y_3 = 0$ $\Rightarrow y_3 = 0$
3. 2nd dual tight or $x_2 = 0$ (Already satisfied)
4. 3rd dual tight or $x_3 = 0$ $\Rightarrow y_2 + 5y_3 = -1$

So, constraints on dual are:

$$\begin{array}{rcl}
y_1 + y_2 & = & 5 \\
3y_2 + 5y_3 & = & -1 \\
y_3 & = & 0
\end{array} \quad \Rightarrow \quad y_2 = -1/3 \quad \Rightarrow \quad y_1 = 5\tfrac{1}{3}$$

Claim: $(5\frac{1}{3}, -\frac{1}{3}, 0)$ is an optimal dual solution.
Proof: Feasible ✓ Obj value is $16 - 8/3 = 15 - 5/3$ $\square$