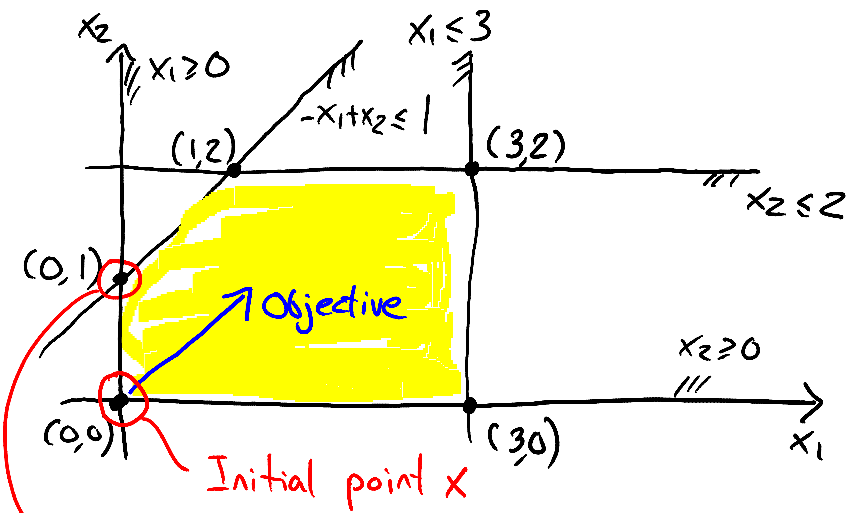
Algorithm Example

Tuesday, September 29, 2009
9:16 AM

$$\begin{array}{lrcl}
\text{Max} & x_1 + x_2 & & \\
\text{s.t.} & -x_1 + x_2 & \le & 1 \\
 & x_1 & \le & 3 \\
 & x_2 & \le & 2 \\
 & x_1, x_2 & \ge & 0
\end{array}$$



Second point $y(\delta)$ (See calculation below)

Convert to Equational Form

$$\begin{array}{lrcl}
\text{Max} & x_1 + x_2 & & \\
\text{s.t.} & -x_1 + x_2 + x_3 & = & 1 \\
 & x_1 + x_4 & = & 3 \\
 & x_2 + x_5 & = & 2 \\
 & x \ge 0 & &
\end{array}$$

$\underbrace{x_3, x_4, x_5}_{\text{Slack Variables}}$

Finding starting feasible basis

We don't know how... just do it by inspection

$B = \{3, 4, 5\}$ a feasible basis

$x = [0, 0, 1, 3, 2]$ Obj fun = 0

Need to choose "entering coordinate"

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What is benefit of x_1?

$d_B = -A_B^{-1} A_1 \quad d_1 = 1 \quad d_2 = 0$

$d = [1 \; 0 \; 1 \; -1 \; 0]$

$c^T d = 1$ This is benefit of x_1

Benefit of x_2?

$d_B = -A_B^{-1} A_2 \quad d_2 = 1 \quad d_1 = 0$

$d = [0 \; 1 \; -1 \; 0 \; -1]$

$c^T d = 1$ This is benefit of x_2

Could increase either x_1 or x_2. We don't care which

Increase x_2 by ε

Move to the new point $y(\varepsilon)$

$y(\varepsilon) = x + \varepsilon d$

$$\delta = \min \left\{ \frac{-x_i}{d_i} : i \text{ s.t. } d_i < 0 \right\}$$

$$= \min \left\{ \frac{-x_3}{d_{\boxed{3}}}, \frac{-x_5}{d_5} \right\} = \min\{1, 2\} = 1$$

$\delta = 1 \qquad \boxed{h = 3}$ 3 is the "leaving coordinate"

Our new BFS is

$y(\delta) = [0 \; 1 \; 0 \; 3 \; 1]$

Note: $c^T y(\delta) = 1 > c^T x = 0$

So new BFS is better

Can continue this process to find optimal point.