

University of Waterloo
PMATH 445/745 SAMPLE Test 2
Winter 2025

Date: Wednesday, Mar 19, 2025
Exam period: 4:30 PM to 5:20 PM
Duration: 50 minutes

Additional instructions

1. Write your answers in the space provided. If you need more room, use the space available on the last page, and indicate this on the question page where you ran out of room.
2. *Do NOT detach any paper from this test.*
3. There are 3 problems worth a total of 17 marks.

Q1. (5 marks) Short answer; no justification required. 1 mark each.

(a) A certain group G of order 16 has $[G, G] \cong C_2$. How many rows of the character table of G have a 1 in the first column?

(b) What is the number of standard Young tableaux of shape $(3, 2, 2)$?

(c) Let $T = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 4 \\ \hline 5 & 6 & & \\ \hline \end{array}$. Give an example of a non-identity element that belongs to $C(T)$.

(d) Given an example of a simple $M_2(\mathbb{R})$ -module.

(e) Give an example of rings $S \subseteq R$ such that R is semisimple but S is not.

Q2. (8 marks) Short answer; give brief justification. 2 marks each.

(a) Complete the character table below, which is missing some rows.

Note: $\omega = e^{2\pi i/3}$.

	1	4	4	3
	C_0	C_1	C_2	C_3
χ_0	1	1	1	1
χ_1	1	ω	ω^2	1

(b) Find the character values of $\chi_{(3,1)}$ and χ_S , where $\chi_{(3,1)}$ is the character of the Specht module $S_{(3,1)}$ of S_4 and χ_S is the character of $\text{Sym}^2(S_{(3,1)})$.

	e	(1 2)	(1 2 3)	(1 2 3 4)	(1 2)(3 4)
$\chi_{(3,1)}$					
χ_S					

(c) Show that $\mathbb{R}[x]/(x^2)$ is not a semisimple ring.

(d) Show, using the definition of the Specht module, that $S_{(1,1,1)}$ is isomorphic to the alternating representation of S_3 .

Q3. (4 marks) Let I_1, \dots, I_k be left ideals in R . Show that ${}_R R = I_1 \oplus \dots \oplus I_k$ if and only if there exist $e_1, \dots, e_k \in R$ that satisfy:

- $e_1 + \dots + e_k = 1$.
- $e_i^2 = e_i$ for all i and $e_i e_j = 0$ for $i \neq j$.

Furthermore, show that in this case $I_i = R e_i$ for all i .

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You may use this space if you run out of room for a particular question.
If you do, be sure to indicate this clearly here on this page and also on the question
page.
