Mathematical Analysis of “phase ramping” for Super-resolution MRI

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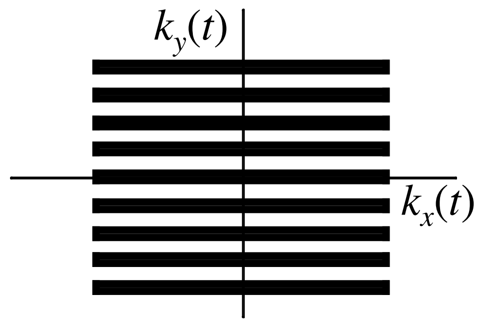
MRI Data is Acquired in the Frequency Domain



$$L_1(k) = \text{III}\left(\frac{k}{\Delta k}\right) \int_{-k_{max}}^{k_{max}} s(\kappa)\psi(k-\kappa)d\kappa$$

- data is briefly continous along $k_x(t)$ before it is sampled
- $k_x(t)$is called the "frequency encoding" direction
- image is obtained with an inverse discrete Fourier transform

Frequency Information in MRI

$$\Delta x = 1/(2k_{max})$$

Frequency domain (log magnitude)

1×

4×

64×

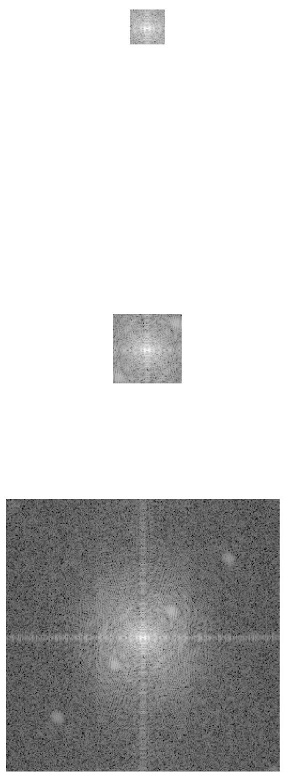
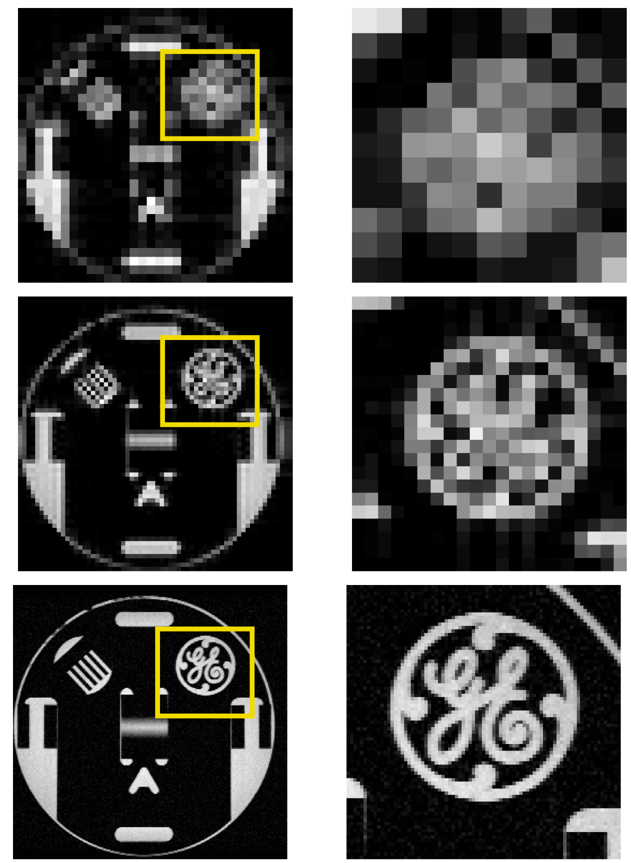
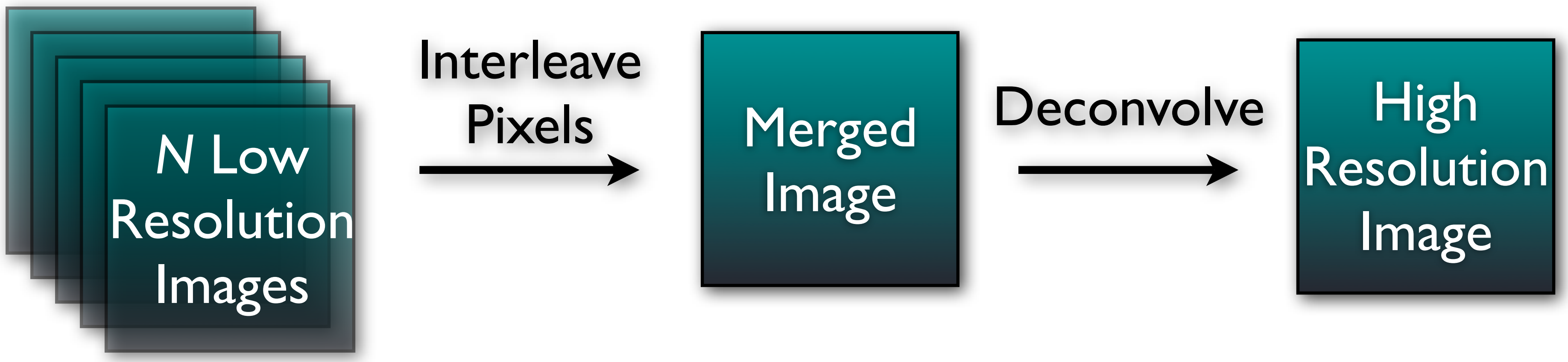


Image Close-up



Multiple Acquisition Super-Resolution Methods



- very little work on this approach in MRI
- field of view is shifted between *acquisitions*
- shifts are distances **<u>less than a pixel</u>** spacing, Δx or Δy

Initial Work on Super-Resolution MRI

first super-resolution MRI paper published in 2001

Peled S, Yeshurun Y, Magnetic Resonance in Medicine, 45, 29-35, 2001

this contribution was received with controversy:

> " ... no new information can be acquired using different in-plane shifts since an in-plane shift is a postprocessing step ... "

> "It is, therefore, not possible to increase resolution by acquiring images that are spatially shifted against each other ... "

Scheffler, K, Magnetic Resonance in Medicine, 48, 408, 2002

The Fourier Shift Theorem and Super-Resolution MRI

- the Fourier shift theorem simply states, for some constant r_n:

$$R(m\Delta k, r_n)\, F(k_x, k_y) \longrightarrow f(x - r_n, y)$$

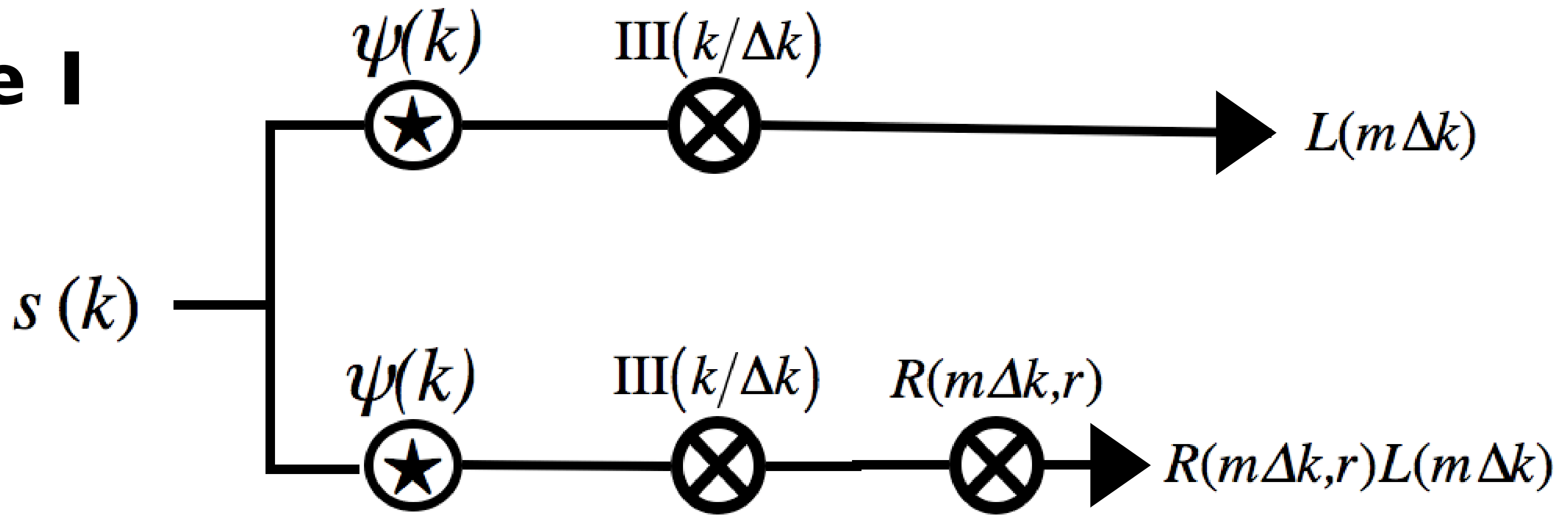
- $R(m\Delta k, r_n) = \exp(-2\pi i r_n k_x)$ is the "phase ramp"
- Scheffler argued: acquire only one image, and use phase ramps to produce the other images
- in this scenario: there is no new information in each image after the first, and no apparent advantage to using multiple data sets

however ....

Phase Ramp Applied After Sampling

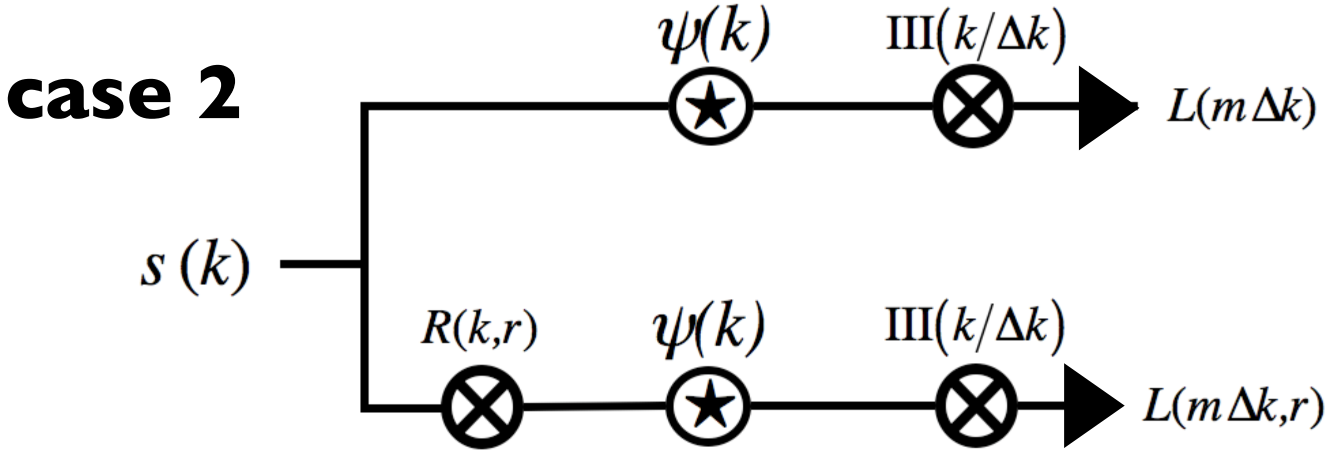
- can the phase ramp be applied in such a way that new information is present in each acquisition?

case 1



the phase ramp, *R*, does not add new information to the signal

Phase Ramp Applied Before Low-Pass Filter



- new information is present because integration and multiplication do not commute
- the recent MRI literature has noted that this could add new information in L(mΔk,r)

Our Research

- a study on **<u>how much</u>** new information is present in each acquisition is absent in the MRI literature
- hence, our work is focused on introducing measures to characterize the new information is present in each acquisition after the first
- we first used the ordinary dot product between two vectors:

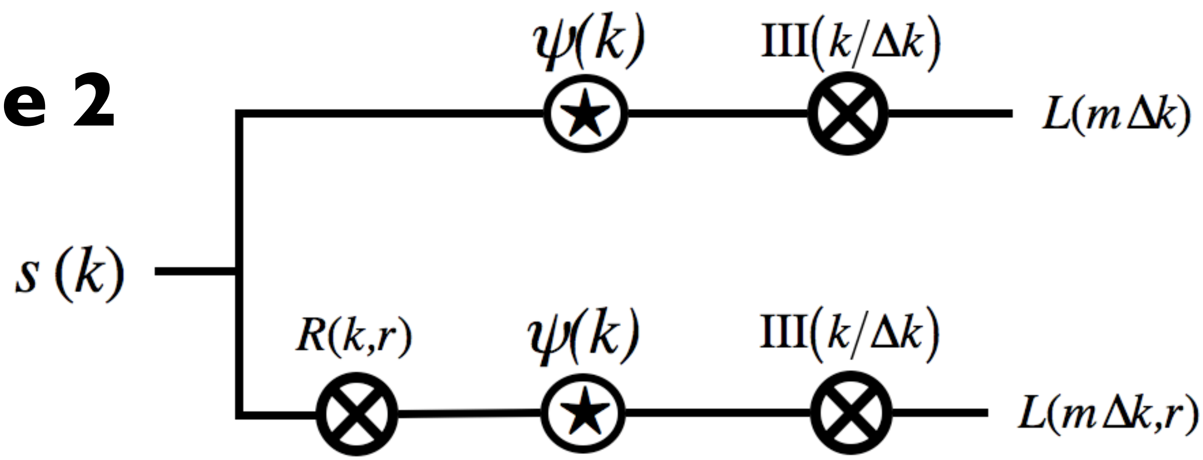
$$C = \cos\theta = \frac{\bar{a} \cdot b}{|a||b|}$$

- if C = 0, a and b are orthogonal
- if C is close to one, a and b are "highly correlated"

Simulation 1

we can study the relative information between signals with different phase ramps

case 2



we will take the dot product between these two signals for a range of r values

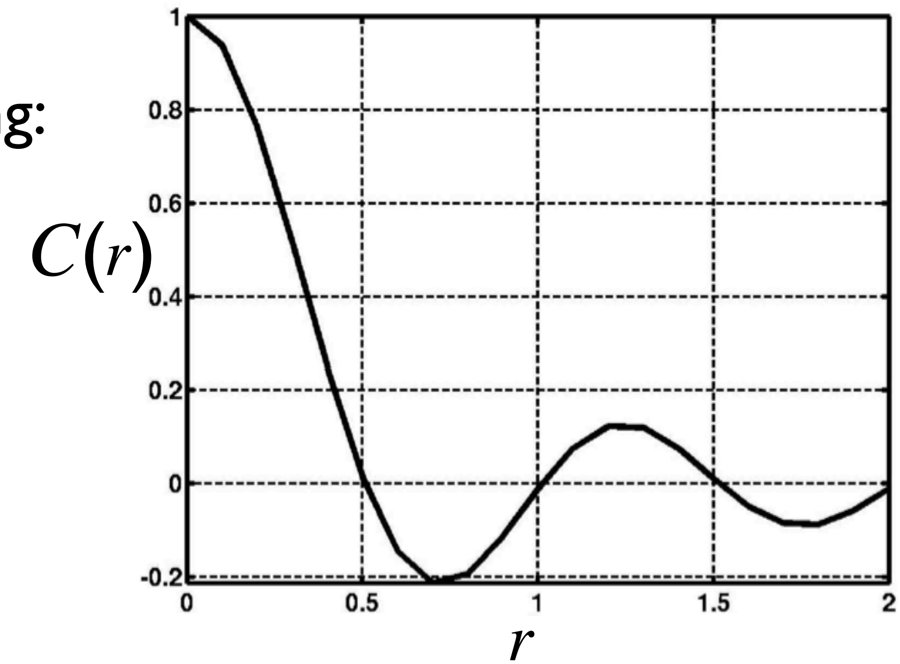
Simulation 1

a simulation was performed using:

$$s(k) = \mathrm{sinc}^2(10\Delta x k)$$

$\Delta x = 1/2$

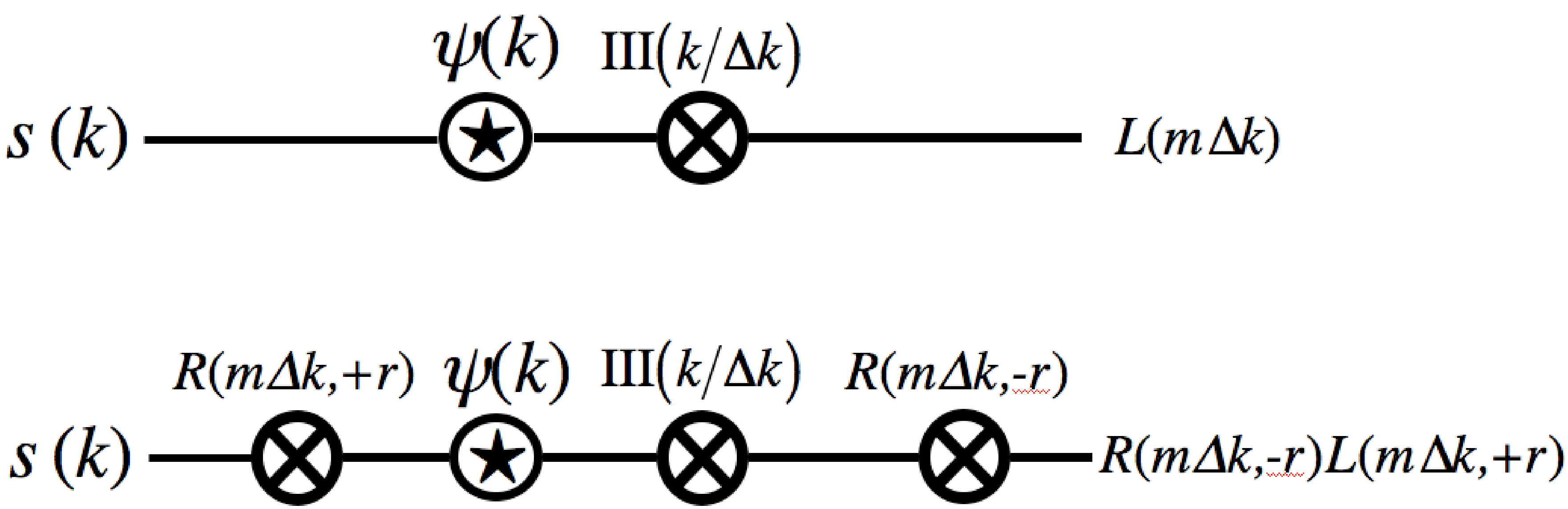
$k \in [-1,1]$



- numerical integration (Simpson's rule) was used to simulate the convolution integral
- as r was increased, the two vectors became less correlated!
- <u>initially</u>, this suggested that a significant amount of new information was present in each data set for increasing r

Simulation 2

we studied the extent to which we can undo the phase ramp with a second phase ramp



we calculated: $C_R = L(m\Delta k) \bullet \left[R(m\Delta k,-r) L(m\Delta k,+r) \right]$

Simulation 2

simulations were performed using realistic MRI data acquisition parameters

$$s(k_m) = w \sum_{p=1}^{10} \frac{\sin^2(w\pi k_m)}{(w\pi k_m)^2} \exp(-2\pi i k_m d_p)$$

$d_p = (-.5 + p4/9)$ cm, $p = 0,1,2, \ldots 9$

$w = 0.1$ cm

$k_m = m\Delta k = \gamma G(m\text{-}M/2)\Delta t$

$\gamma = 42.58$ MHz/T

$G = 0.01$ T/cm

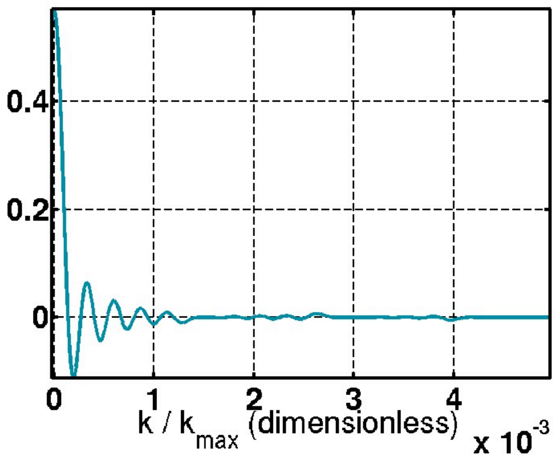
$M = 100000$

$\Delta t = 10^{-6}$ s

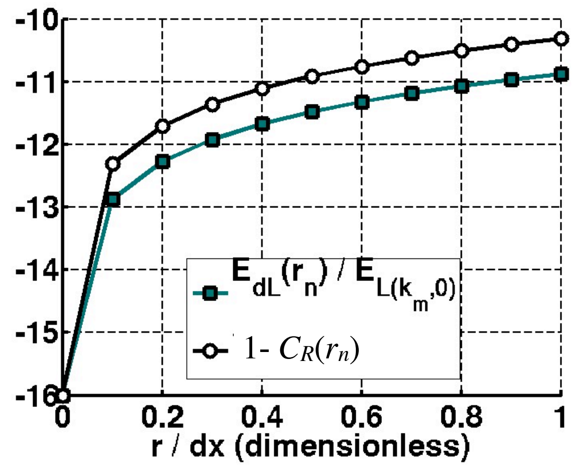
numerical integration (64 term Gaussian quadrature scheme) was used to simulate the convolution integral

Simulation 2 Results

simulated data



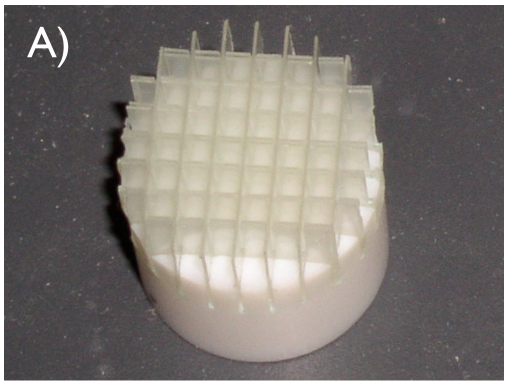
results for different r_n values



$C_R(r_n)$ was close to one:

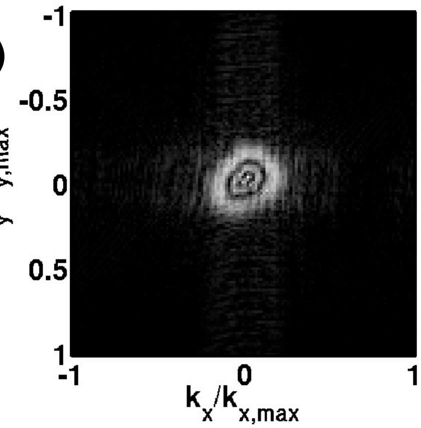
- the two vectors were highly correlated
- there is some information present in each acqusition
- the relative amount of new information in each acquisition is small ($C_R(r_n) < 10^{-10}$)

Experiment with MRI Data



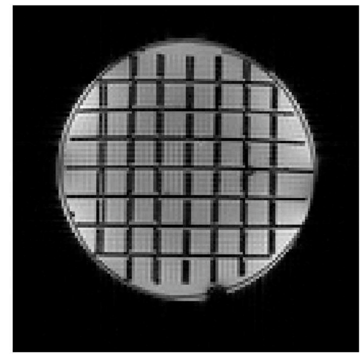
A photograph of the phantom consisting of plastic slats mounted on a 2 cm diameter teflon base

B)

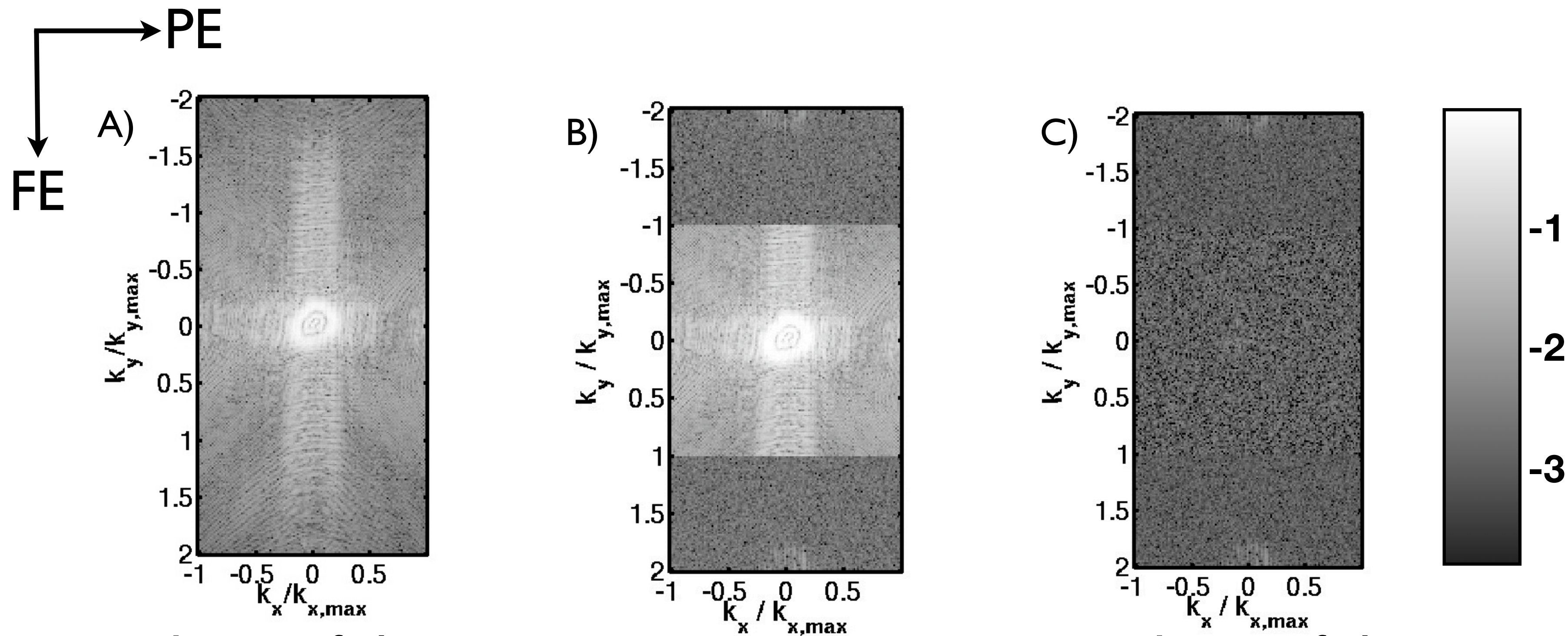


The amplitude spectrum of the measured signal from the phantom.

C)



MR image of the unshifted phantom immersed in water.



$\log_{10}$ of the amplitude spectrum of the measured high resolution data

$\log_{10}$ of the amplitude spectrum of the merged data.

$\log_{10}$ of the difference between a zero padded version of $L(k_x,k_y,0)$ and the merged data.

the merged data can be approximated by zero padding a single low resolution image

Conclusions

- new information can be present in each acquisition after the first
- the amount of new information is relatively small
- it remains to be shown how this small amount of information can be used to improve the spatial resolution
- it is not clear how SR MRI can compete with more established resolution enhancement strategies that emply only one image
- future research on SR MRI could focus on spatial shifts out of the image plane