C&O 330 Combinatorial Enumeration

Instructor: Professor D.M.Jackson

Autumn 2004

I **Organisation:**

Assignments [15%]: Due on Friday at the start of class. There will be one every two weeks. Rather than distributing paper copies of the assignments, I shall put .pdf copies of the assignments and their solutions on this web site.

Midterm [25%]: Time to be arranged.

Final [60%]: 3 hours. Date is scheduled by the Registrar.

II **Background:** Discrete structures occur in a wide range of applications. The ones that are probably most familiar to you are from computer science and include trees, stacks and paths. There are some highly significant applications to physics (paths between parallel planes for the computation of the binding energy of polymers: embeddings of graphs in orientable surfaces for determining the energy function in models from quantum gravity), to genetics (the analysis of DNA strings) and to other parts of mathematics. Instances of the latter are discussed in C&O 430/630.

In this course we shall develop and use some of the tools that allow us to decompose, construct and count discrete structures, and we will develop some general approaches for classes of structures. Often a complex combinatorial structure can be reduced to one of the classical ones by an appropriate algorithm. This accounts for our interest in examining a number of the classical structures in some depth.

The core courses that you have been taking in this Faculty provide the tools for extracting combinatorial information from discrete structures. This may come as a surprise, for it was certainly not the reason for the historical development of areas such as linear algebra, analysis, and algebra (or even group theory and topology). However, it does say something potent about the fundamental nature of the ideas behind this area of combinatorics and its connexion with these areas of mathematics. A certain amount of linear algebra and matrix theory will be used in this course.

III **Topics covered:** These will be selected from the following list.

0 The combinatorics of the ordinary generating series: Review of the ordinary generating series, and some new bijections. Partitions of integers, the classical identities of Gauss, Cauchy and Jacobi;

1 Sign reversing involutions: Ordered sets of non-intersecting paths on the integer points of the plane, lattice polygons, Young tableaux;

2 Sequences: Application of some elementary linear algebra to count sequences over a finite alphabet, with conditions placed on adjacent symbols;

3 The combinatorics of the exponential generating series: Permutations, trees, Lagrange's inversion theorem, self-nonintersecting paths in the plane;

4 Structures with non-trivial automorphisms: The Principle of Inclusion and Exclusion; Polya's theorem, graph counting.

IV **Assumptions:** I shall assume that you are familiar with the elementary expansion of formal power series. This was covered at length in Math 239, and you should consult the notes for that course if you need to reacquaint yourself with the expansions.

V **Notes:** Notes for this course will available from the Copy Centre in this building soon after the beginning of term (price to be determined). They are typeset (Latex) and bound. The topics covered in class will be a subset selected from the notes, but you are strongly urged to read through the topics that are not covered in class, to gain a broader perspective of the area.