C&O 330 - ASSIGNMENT #1

DUE FRIDAY, 1 OCTOBER AT 10:31PM

(1) **(20 points)** Give an expression for $[x^n]\,G$ as an explicit function of n of each of the following two functional equations.

(a) **(10 points)** $G = F$, where F satisfies the functional equation

$$F = \frac{x}{(1-F)^m},$$

where m is a positive integer.

(b) **(10 points)** $G = e^T$ where T satisfies the functional equation

$$T = xe^T.$$

(2) **(20 points)** A regular n-gon (that is, a regular polygon with n sides) A in the plane has a distinguished edge. A *diagonal* of A is a line segment, in the interior of A, joining two vertices of A. Find the number, c_n, of dissections of A into triangles by diagonals that meet only at vertices of A.

(3) **(20 points)** Find the number, a_n, of plane planted trees on n non-root vertices with no vertices of degree 2. Your answer should give a_n as an explicit function of n.

(4) **(20 points)**

(a) **(10 points)** Solve the functional equation

$$P = x\left(\frac{1}{1-P} + (u-1)\,P\right),$$

for $P \equiv P(x,u)$, where x and u are indeterminates, by giving $\left[x^n u^k\right] P(x,u)$ as an explicit expression in n and k.

(b) **(10 points)** Construct a set $\mathcal{P}$ of *combinatorial structures* and a weight function ω for $\mathcal{P}$ such that

$$\left[(\mathcal{P},\omega)\right]_o = P(x,u).$$

Note that the weight function ω must be of the form

$$\omega\colon \mathcal{P} \to \{0,1,2,\ldots\}^2 : \sigma \mapsto (\omega_1(\sigma), \omega_2(\sigma))$$

where

$$\omega_1, \omega_2\colon \mathcal{P} \to \{0,1,2,\ldots\}.$$

and the form of the generating series P is

$$P(x,u) = \sum_{\sigma\in\mathcal{P}} x^{\omega_1(\sigma)} u^{\omega_2(\sigma)}.$$

(**Remark:** You have been introduced to bivariate generating series (generating series in two indeterminates) in Math 239.)