

# GENERATORS OF SUBCONES OF THE NEF CONE OF A CUBIC SURFACE

All of these tables were generated using the PORTA software package, written by Thomas Christof and Andreas Loebel in 1997.

We begin by defining the notation used in these tables. Let  $\phi: X \rightarrow \mathbb{P}^2$  be the blowup of  $\mathbb{P}^2$  at six rational points in general position, and let  $E_1, \dots, E_6$  be the exceptional divisors of  $\phi$ . We define the following linear equivalence classes on  $X$ :

- $L = \phi^* \mathcal{O}(1)$
- $L_i = L - E_i$ , the strict transform of a line through  $P_i = \phi(E_i)$
- $L_{ij} = 2L - (\sum E_n) + E_i + E_j$ , the strict transform of a conic through the four points  $P_n$  with  $n \neq i, j$
- $B_i = 3L - (\sum E_n) - E_i$ , the strict transform of a cubic curve through all six points  $P_n$ , with a node at  $P_i$ .

Let  $h$  be the class of a hyperplane in the embedding  $X \subset \mathbb{P}^3$ . For any line  $\ell$  on  $X$ , the hyperplanes containing  $\ell$  give (after removing  $\ell$ ) a base-point-free pencil on  $X$ . The classes  $\{L_i, L_{ij}, B_i\}$  defined above are the 27 pencils coming from the lines. In addition, for any point  $x$  on  $X$  let  $C_x$  be the intersection of  $X$  with its tangent plane at  $x$  (so  $C_x$  has class  $h$ ). If  $x$  does not lie on a line, then  $C_x$  is a plane cubic curve with one double point, at  $x$ .

Let  $\Gamma$  be the nef cone of  $X$ , and  $S$  be the set of 27 divisor classes  $\{L_i, L_{ij}, B_i\}$  as  $i$  and  $j$  range over all possible values. For each element  $C$  in  $S$ , we define the subcone  $\Gamma(C)$  by:

$$\Gamma(C) = \left\{ D \in \Gamma \mid D.C = \min_{C' \in S} \{D.C'\} \text{ and } D.C \leq (D.h)/2 \right\}.$$

Further define the subcone  $\Gamma(h)$  to be:

$$\Gamma(h) = \left\{ D \in \Gamma \mid (D.h)/2 \leq \min_{C' \in S} \{D.C'\} \right\}.$$

It is clear that  $\Gamma$  is the union of these 28 subcones.

In each of the tables in this file, the first column is a numerical identifier of the vector in that row. The subsequent columns represent the coefficients of the vector with respect to the basis  $\{L, E_1, \dots, E_6\}$  of the Néron-Severi group of  $X$ . Thus, vector number 1 in Table 1 is the divisor class  $2L - E_1 - E_2 - E_3$ . Each of the cones has 99 generators. There is no correspondence or relation between rows in different tables with the same numerical identifier.

Table 1, of generators of the nef cone, is reproducing information that has been well known for some time, of course. It was calculated for these tables by finding generators for the cone obtained as the intersection of the half-spaces corresponding to non-negative intersection with each of the 27 lines on the cubic surface. The other tables were generated in a similar way. For instance, Table 2, of generators of the cone  $\Gamma(L_1)$ , was generated by using the half-spaces defining  $\Gamma$ , in addition to the half-spaces corresponding to the intersection inequalities described above.

Table 1: Generators of the nef cone  $\Gamma$  of a smooth cubic surface

| #  | $L$ | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ | $E_6$ | #  | $L$ | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ | $E_6$ | #  | $L$ | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ | $E_6$ |
|----|-----|-------|-------|-------|-------|-------|-------|----|-----|-------|-------|-------|-------|-------|-------|----|-----|-------|-------|-------|-------|-------|-------|
| 1  | 2   | -1    | -1    | -1    | 0     | 0     | 0     | 34 | 2   | -1    | -1    | -1    | 0     | -1    | 0     | 67 | 3   | -1    | 0     | -1    | -1    | -1    | -2    |
| 2  | 2   | -1    | -1    | 0     | -1    | 0     | 0     | 35 | 2   | -1    | -1    | -1    | 0     | 0     | -1    | 68 | 3   | 0     | -2    | -1    | -1    | -1    | -1    |
| 3  | 2   | -1    | -1    | 0     | 0     | -1    | 0     | 36 | 2   | -1    | -1    | 0     | -1    | -1    | 0     | 69 | 3   | 0     | -1    | -2    | -1    | -1    | -1    |
| 4  | 2   | -1    | -1    | 0     | 0     | 0     | -1    | 37 | 2   | -1    | -1    | 0     | -1    | 0     | -1    | 70 | 3   | 0     | -1    | -1    | -2    | -1    | -1    |
| 5  | 2   | -1    | 0     | -1    | -1    | 0     | 0     | 38 | 2   | -1    | -1    | 0     | 0     | -1    | -1    | 71 | 3   | 0     | -1    | -1    | -1    | -2    | -1    |
| 6  | 2   | -1    | 0     | -1    | 0     | -1    | 0     | 39 | 2   | -1    | 0     | -1    | -1    | -1    | 0     | 72 | 3   | 0     | -1    | -1    | -1    | -1    | -2    |
| 7  | 2   | -1    | 0     | -1    | 0     | 0     | -1    | 40 | 2   | -1    | 0     | -1    | -1    | 0     | -1    | 73 | 3   | -2    | -1    | -1    | -1    | -1    | -1    |
| 8  | 2   | -1    | 0     | 0     | -1    | -1    | 0     | 41 | 2   | -1    | 0     | -1    | 0     | -1    | -1    | 74 | 3   | -1    | -2    | -1    | -1    | -1    | -1    |
| 9  | 2   | -1    | 0     | 0     | -1    | 0     | -1    | 42 | 2   | -1    | 0     | 0     | -1    | -1    | -1    | 75 | 3   | -1    | -1    | -2    | -1    | -1    | -1    |
| 10 | 2   | -1    | 0     | 0     | 0     | -1    | -1    | 43 | 2   | 0     | -1    | -1    | -1    | -1    | 0     | 76 | 3   | -1    | -1    | -1    | -2    | -1    | -1    |
| 11 | 1   | 0     | 0     | 0     | 0     | 0     | 0     | 44 | 2   | 0     | -1    | -1    | -1    | 0     | -1    | 77 | 3   | -1    | -1    | -1    | -1    | -2    | -1    |
| 12 | 3   | -2    | -1    | -1    | -1    | -1    | 0     | 45 | 2   | 0     | -1    | -1    | 0     | -1    | -1    | 78 | 3   | -1    | -1    | -1    | -1    | -1    | -2    |
| 13 | 3   | -2    | -1    | -1    | -1    | 0     | -1    | 46 | 2   | 0     | -1    | 0     | -1    | -1    | -1    | 79 | 4   | -2    | -2    | -2    | -1    | -1    | -1    |
| 14 | 3   | -2    | -1    | -1    | 0     | -1    | -1    | 47 | 2   | 0     | 0     | -1    | -1    | -1    | -1    | 80 | 4   | -2    | -2    | -1    | -2    | -1    | -1    |
| 15 | 3   | -2    | -1    | 0     | -1    | -1    | -1    | 48 | 3   | -1    | -2    | -1    | -1    | -1    | 0     | 81 | 4   | -2    | -2    | -1    | -1    | -2    | -1    |
| 16 | 3   | -2    | 0     | -1    | -1    | -1    | -1    | 49 | 3   | -1    | -2    | -1    | -1    | 0     | -1    | 82 | 4   | -2    | -2    | -1    | -1    | -1    | -2    |
| 17 | 1   | -1    | 0     | 0     | 0     | 0     | 0     | 50 | 3   | -1    | -2    | -1    | 0     | -1    | -1    | 83 | 4   | -2    | -1    | -2    | -2    | -1    | -1    |
| 18 | 1   | 0     | -1    | 0     | 0     | 0     | 0     | 51 | 3   | -1    | -2    | 0     | -1    | -1    | -1    | 84 | 4   | -2    | -1    | -2    | -1    | -2    | -1    |
| 19 | 1   | 0     | 0     | -1    | 0     | 0     | 0     | 52 | 3   | -1    | -1    | -2    | -1    | -1    | 0     | 85 | 4   | -2    | -1    | -2    | -1    | -1    | -2    |
| 20 | 1   | 0     | 0     | 0     | -1    | 0     | 0     | 53 | 3   | -1    | -1    | -2    | -1    | 0     | -1    | 86 | 4   | -2    | -1    | -1    | -2    | -2    | -1    |
| 21 | 1   | 0     | 0     | 0     | 0     | -1    | 0     | 54 | 3   | -1    | -1    | -2    | 0     | -1    | -1    | 87 | 4   | -2    | -1    | -1    | -2    | -1    | -2    |
| 22 | 1   | 0     | 0     | 0     | 0     | 0     | -1    | 55 | 3   | -1    | -1    | -1    | -2    | -1    | 0     | 88 | 4   | -2    | -1    | -1    | -1    | -2    | -2    |
| 23 | 2   | 0     | -1    | -1    | -1    | 0     | 0     | 56 | 3   | -1    | -1    | -1    | -2    | 0     | -1    | 89 | 4   | -1    | -2    | -2    | -2    | -1    | -1    |
| 24 | 2   | 0     | -1    | -1    | 0     | -1    | 0     | 57 | 3   | -1    | -1    | -1    | -1    | -2    | 0     | 90 | 4   | -1    | -2    | -2    | -1    | -2    | -1    |
| 25 | 2   | 0     | -1    | -1    | 0     | 0     | -1    | 58 | 3   | -1    | -1    | -1    | -1    | 0     | -2    | 91 | 4   | -1    | -2    | -2    | -1    | -1    | -2    |
| 26 | 2   | 0     | -1    | 0     | -1    | -1    | 0     | 59 | 3   | -1    | -1    | -1    | 0     | -2    | -1    | 92 | 4   | -1    | -2    | -1    | -2    | -2    | -1    |
| 27 | 2   | 0     | -1    | 0     | -1    | 0     | -1    | 60 | 3   | -1    | -1    | -1    | 0     | -1    | -2    | 93 | 4   | -1    | -2    | -1    | -2    | -1    | -2    |
| 28 | 2   | 0     | -1    | 0     | 0     | -1    | -1    | 61 | 3   | -1    | -1    | 0     | -2    | -1    | -1    | 94 | 4   | -1    | -2    | -1    | -1    | -2    | -2    |
| 29 | 2   | 0     | 0     | -1    | -1    | -1    | 0     | 62 | 3   | -1    | -1    | 0     | -1    | -2    | -1    | 95 | 4   | -1    | -1    | -2    | -2    | -2    | -1    |
| 30 | 2   | 0     | 0     | -1    | -1    | 0     | -1    | 63 | 3   | -1    | -1    | 0     | -1    | -1    | -2    | 96 | 4   | -1    | -1    | -2    | -2    | -1    | -2    |
| 31 | 2   | 0     | 0     | -1    | 0     | -1    | -1    | 64 | 3   | -1    | 0     | -2    | -1    | -1    | -1    | 97 | 4   | -1    | -1    | -2    | -1    | -2    | -2    |
| 32 | 2   | 0     | 0     | 0     | -1    | -1    | -1    | 65 | 3   | -1    | 0     | -1    | -2    | -1    | -1    | 98 | 4   | -1    | -1    | -1    | -2    | -2    | -2    |
| 33 | 2   | -1    | -1    | -1    | -1    | 0     | 0     | 66 | 3   | -1    | 0     | -1    | -1    | -2    | -1    | 99 | 5   | -2    | -2    | -2    | -2    | -2    | -2    |

**Remarks on other tables.** In Table 2 which follows, we use  $D_n$  to refer to the divisor class represented by row  $n$  of Table 2. For any point  $x \in X$  not on a  $(-1)$ -curve, the unique curve  $F = F_{x,L_1}$  in the pencil  $L_1$  passing through  $x$  is smooth and irreducible. In each line of the table “Reason” is a — very brief! — justification of why  $F$  is a Seshadri curve for  $x$  with respect to  $D_n$ .

For instance, in row 1 of Table 2, the “Reason” is  $L_1.D_1 = 1$ , and thus  $F.D_1 = L_1.D_1 = 1$ . We claim that for the divisors  $D_i$ ,  $\epsilon_x$  is always at least one if it is nonzero. To see this, notice that the generators of the nef cone (see Table 1) are all either morphisms to  $\mathbb{P}^1$  corresponding to pencils of conics on the cubic surface, or else morphisms to  $\mathbb{P}^2$  that are the blowing down of six pairwise disjoint  $(-1)$ -curves. In both cases, the Seshadri constant is easily seen to be either zero or at least one. It is straightforward to check that all the generators listed in Table 2 are non-negative integer linear combinations of the generators of the nef cones, and therefore (by [1, Proposition 2.11(b)]) enjoy the same property: for any point  $x$ , the Seshadri constant  $\epsilon_x(D_i)$  is either zero or else is at least one.

By assumption,  $x$  does not lie on any  $(-1)$ -curve, which are the only curves contracted by any  $D_i$  (except for  $D_{18} = L_1$ , for which  $\epsilon = 0$  for all points). Therefore, since  $F$  has degree 1 with respect to  $D_1$ ,  $F$  is a Seshadri curve for  $x$  with respect to  $D_1$ .

As a second example, in row 29 of Table 2, the comment “ $L + L_{56}$ ” means that the divisor  $D_{29}$  represented by that row is the sum of  $L$  and  $L_{56}$ . Any curve that has nonzero intersection with  $L$  must have  $L.C/\text{mult}_x(C) \geq 1$ , for any  $x$  not lying on a  $(-1)$ -curve, since  $L$  is an isomorphism away from  $(-1)$ -curves. Similarly, any curve not contracted by  $L_{56}$  must also satisfy  $L_{56}.C/\text{mult}_x(C) \geq 1$ , so any curve not contracted by  $L_{56}$  or  $L$  must satisfy  $(L+L_{56}).C/\text{mult}_x(C) \geq 2$ . If  $C$  is contracted by  $L_{56}$ , then it is either a  $(-1)$ -curve, or else it is an element of the divisor class  $L_{56}$  itself, in which case it satisfies  $(L+L_{56}).C/\text{mult}_x(C) = 2$  by direct calculation. In all cases, since  $x$  does not lie on a  $(-1)$ -curve, we see that  $\epsilon_x(L+L_{56}) \geq 2$ , and since  $L_1.L = L_1.L_{56} = 1$ , we conclude that  $\epsilon_x(L + L_{56}) = 2$ , and so the curve in the class  $L_1$  through  $x$  is a Seshadri curve for  $x$  with respect to  $D_{29} = L + L_{56}$ . Similar arguments explain the other reasons of the form “ $A + B$ ” or “ $A + B + C$ ”.

In light of these arguments, for Table 2, it is useful to know that  $L_1$  has intersection number one with the divisors  $L$ ,  $B_1$ ,  $L_i$  for  $i \neq 1$ , and  $L_{ij}$  for  $i, j \neq 1$ .

In Table 3, the rightmost column of row  $n$  contains a divisor class  $C \in S$  such that  $G_n$  (the divisor corresponding to the  $n$ th row of Table 3) is also a generator of the subcone  $\Gamma(C)$ . From the definition of the cones  $\Gamma(C)$  and  $\Gamma(h)$ , this implies that  $G_n.C = (G_n.h)/2$ . As explained in the proof of [1, Theorem 4.7], this provides a verification that  $C_x$  is a Seshadri curve for  $x$  with respect to  $G_n$ .

Tables 4 and 5 are simply lists of generators of the named subcones. By permuting the indices  $1, \dots, 6$ , Tables 2, 4, and 5 yield generators for all the subcones  $\Gamma(L_i)$ ,  $\Gamma(B_i)$  and  $\Gamma(L_{ij})$ .





Table 4: Generators of the cone  $\Gamma(B_1)$

| #  | L | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ | $E_6$ | #  | L | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ | $E_6$ | #  | L | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ | $E_6$ |
|----|---|-------|-------|-------|-------|-------|-------|----|---|-------|-------|-------|-------|-------|-------|----|---|-------|-------|-------|-------|-------|-------|
| 1  | 4 | -3    | -1    | -1    | -1    | -1    | -1    | 34 | 4 | -2    | -1    | -2    | -1    | -1    | -2    | 67 | 6 | -3    | -2    | -2    | -3    | -1    | -2    |
| 2  | 3 | -2    | -1    | -1    | -1    | -1    | -1    | 35 | 4 | -2    | -1    | -1    | -2    | -2    | -1    | 68 | 6 | -3    | -2    | -2    | -2    | -3    | -1    |
| 3  | 5 | -3    | -2    | -2    | -2    | -1    | -1    | 36 | 4 | -2    | -1    | -1    | -2    | -1    | -2    | 69 | 6 | -3    | -2    | -2    | -2    | -1    | -3    |
| 4  | 5 | -3    | -2    | -2    | -1    | -2    | -1    | 37 | 4 | -2    | -1    | -1    | -1    | -2    | -2    | 70 | 6 | -3    | -2    | -2    | -1    | -3    | -2    |
| 5  | 5 | -3    | -2    | -2    | -1    | -1    | -2    | 38 | 5 | -2    | -2    | -2    | -2    | -2    | -1    | 71 | 6 | -3    | -2    | -2    | -1    | -2    | -3    |
| 6  | 5 | -3    | -2    | -1    | -2    | -2    | -1    | 39 | 5 | -2    | -2    | -2    | -2    | -1    | -2    | 72 | 6 | -3    | -2    | -1    | -3    | -2    | -2    |
| 7  | 5 | -3    | -2    | -1    | -2    | -1    | -2    | 40 | 5 | -2    | -2    | -2    | -1    | -2    | -2    | 73 | 6 | -3    | -2    | -1    | -2    | -3    | -2    |
| 8  | 5 | -3    | -2    | -1    | -1    | -2    | -2    | 41 | 5 | -2    | -2    | -1    | -2    | -2    | -2    | 74 | 6 | -3    | -2    | -1    | -2    | -2    | -3    |
| 9  | 5 | -3    | -1    | -2    | -2    | -2    | -1    | 42 | 5 | -2    | -1    | -2    | -2    | -2    | -2    | 75 | 6 | -3    | -1    | -3    | -2    | -2    | -2    |
| 10 | 5 | -3    | -1    | -2    | -2    | -1    | -2    | 43 | 5 | -2    | -2    | -2    | -2    | -2    | -2    | 76 | 6 | -3    | -1    | -2    | -3    | -2    | -2    |
| 11 | 5 | -3    | -1    | -2    | -1    | -2    | -2    | 44 | 5 | -3    | -2    | -2    | -1    | -1    | -1    | 77 | 6 | -3    | -1    | -2    | -2    | -3    | -2    |
| 12 | 5 | -3    | -1    | -1    | -2    | -2    | -2    | 45 | 5 | -3    | -2    | -1    | -2    | -1    | -1    | 78 | 6 | -3    | -1    | -2    | -2    | -2    | -3    |
| 13 | 6 | -3    | -3    | -2    | -2    | -2    | -2    | 46 | 5 | -3    | -2    | -1    | -1    | -2    | -1    | 79 | 7 | -3    | -3    | -3    | -2    | -2    | -2    |
| 14 | 6 | -3    | -2    | -3    | -2    | -2    | -2    | 47 | 5 | -3    | -2    | -1    | -1    | -1    | -2    | 80 | 7 | -3    | -3    | -2    | -3    | -2    | -2    |
| 15 | 6 | -3    | -2    | -2    | -3    | -2    | -2    | 48 | 5 | -3    | -1    | -2    | -2    | -1    | -1    | 81 | 7 | -3    | -3    | -2    | -2    | -3    | -2    |
| 16 | 6 | -3    | -2    | -2    | -2    | -3    | -2    | 49 | 5 | -3    | -1    | -2    | -1    | -2    | -1    | 82 | 7 | -3    | -3    | -2    | -2    | -2    | -3    |
| 17 | 6 | -3    | -2    | -2    | -2    | -2    | -3    | 50 | 5 | -3    | -1    | -2    | -1    | -1    | -2    | 83 | 7 | -3    | -2    | -3    | -3    | -2    | -2    |
| 18 | 3 | -2    | -1    | -1    | -1    | -1    | 0     | 51 | 5 | -3    | -1    | -1    | -2    | -2    | -1    | 84 | 7 | -3    | -2    | -3    | -2    | -3    | -2    |
| 19 | 3 | -2    | -1    | -1    | -1    | 0     | -1    | 52 | 5 | -3    | -1    | -1    | -2    | -1    | -2    | 85 | 7 | -3    | -2    | -3    | -2    | -2    | -3    |
| 20 | 3 | -2    | -1    | -1    | 0     | -1    | -1    | 53 | 5 | -3    | -1    | -1    | -1    | -2    | -2    | 86 | 7 | -3    | -2    | -2    | -3    | -3    | -2    |
| 21 | 3 | -2    | -1    | 0     | -1    | -1    | -1    | 54 | 6 | -3    | -2    | -2    | -2    | -2    | -1    | 87 | 7 | -3    | -2    | -2    | -3    | -2    | -3    |
| 22 | 3 | -2    | 0     | -1    | -1    | -1    | -1    | 55 | 6 | -3    | -2    | -2    | -2    | -1    | -2    | 88 | 7 | -3    | -2    | -2    | -2    | -3    | -3    |
| 23 | 4 | -2    | -2    | -1    | -1    | -1    | -1    | 56 | 6 | -3    | -2    | -2    | -1    | -2    | -2    | 89 | 7 | -3    | -3    | -3    | -3    | -2    | -2    |
| 24 | 4 | -2    | -1    | -2    | -1    | -1    | -1    | 57 | 6 | -3    | -2    | -1    | -2    | -2    | -2    | 90 | 7 | -3    | -3    | -3    | -2    | -3    | -2    |
| 25 | 4 | -2    | -1    | -1    | -2    | -1    | -1    | 58 | 6 | -3    | -1    | -2    | -2    | -2    | -2    | 91 | 7 | -3    | -3    | -3    | -2    | -2    | -3    |
| 26 | 4 | -2    | -1    | -1    | -1    | -2    | -1    | 59 | 6 | -3    | -3    | -2    | -2    | -2    | -1    | 92 | 7 | -3    | -3    | -2    | -3    | -3    | -2    |
| 27 | 4 | -2    | -1    | -1    | -1    | -1    | -2    | 60 | 6 | -3    | -3    | -2    | -2    | -1    | -2    | 93 | 7 | -3    | -3    | -2    | -3    | -2    | -3    |
| 28 | 4 | -2    | -2    | -2    | -1    | -1    | -1    | 61 | 6 | -3    | -3    | -2    | -1    | -2    | -2    | 94 | 7 | -3    | -3    | -2    | -2    | -3    | -3    |
| 29 | 4 | -2    | -2    | -1    | -2    | -1    | -1    | 62 | 6 | -3    | -3    | -1    | -2    | -2    | -2    | 95 | 7 | -3    | -2    | -3    | -3    | -3    | -2    |
| 30 | 4 | -2    | -2    | -1    | -1    | -2    | -1    | 63 | 6 | -3    | -2    | -3    | -2    | -2    | -1    | 96 | 7 | -3    | -2    | -3    | -3    | -2    | -3    |
| 31 | 4 | -2    | -2    | -1    | -1    | -1    | -2    | 64 | 6 | -3    | -2    | -3    | -2    | -1    | -2    | 97 | 7 | -3    | -2    | -3    | -2    | -3    | -3    |
| 32 | 4 | -2    | -1    | -2    | -2    | -1    | -1    | 65 | 6 | -3    | -2    | -3    | -1    | -2    | -2    | 98 | 7 | -3    | -2    | -2    | -3    | -3    | -3    |
| 33 | 4 | -2    | -1    | -2    | -1    | -2    | -1    | 66 | 6 | -3    | -2    | -2    | -3    | -2    | -1    | 99 | 8 | -3    | -3    | -3    | -3    | -3    | -3    |

Table 5: Generators of the cone  $\Gamma(L_{56})$

| #  | L | E <sub>1</sub> | E <sub>2</sub> | E <sub>3</sub> | E <sub>4</sub> | E <sub>5</sub> | E <sub>6</sub> | #  | L | E <sub>1</sub> | E <sub>2</sub> | E <sub>3</sub> | E <sub>4</sub> | E <sub>5</sub> | E <sub>6</sub> | #  | L | E <sub>1</sub> | E <sub>2</sub> | E <sub>3</sub> | E <sub>4</sub> | E <sub>5</sub> | E <sub>6</sub> |
|----|---|----------------|----------------|----------------|----------------|----------------|----------------|----|---|----------------|----------------|----------------|----------------|----------------|----------------|----|---|----------------|----------------|----------------|----------------|----------------|----------------|
| 1  | 2 | -1             | -1             | -1             | -1             | 0              | 0              | 34 | 4 | -2             | -2             | -1             | -1             | 0              | -1             | 67 | 5 | -2             | -3             | -1             | -2             | -1             | -1             |
| 2  | 3 | -2             | -1             | -1             | -1             | 0              | 0              | 35 | 4 | -2             | -1             | -2             | -1             | -1             | 0              | 68 | 5 | -2             | -2             | -3             | -1             | -1             | -1             |
| 3  | 3 | -1             | -2             | -1             | -1             | 0              | 0              | 36 | 4 | -2             | -1             | -2             | -1             | 0              | -1             | 69 | 5 | -2             | -2             | -1             | -3             | -1             | -1             |
| 4  | 3 | -1             | -1             | -2             | -1             | 0              | 0              | 37 | 4 | -2             | -1             | -1             | -2             | -1             | 0              | 70 | 5 | -2             | -1             | -3             | -2             | -1             | -1             |
| 5  | 3 | -1             | -1             | -1             | -2             | 0              | 0              | 38 | 4 | -2             | -1             | -1             | -2             | 0              | -1             | 71 | 5 | -2             | -1             | -2             | -3             | -1             | -1             |
| 6  | 5 | -3             | -2             | -2             | -2             | -1             | -1             | 39 | 4 | -1             | -2             | -2             | -1             | -1             | 0              | 72 | 5 | -1             | -3             | -2             | -2             | -1             | -1             |
| 7  | 5 | -2             | -3             | -2             | -2             | -1             | -1             | 40 | 4 | -1             | -2             | -2             | -1             | 0              | -1             | 73 | 5 | -1             | -2             | -3             | -2             | -1             | -1             |
| 8  | 5 | -2             | -2             | -3             | -2             | -1             | -1             | 41 | 4 | -1             | -2             | -1             | -2             | -1             | 0              | 74 | 5 | -1             | -2             | -2             | -3             | -1             | -1             |
| 9  | 5 | -2             | -2             | -2             | -3             | -1             | -1             | 42 | 4 | -1             | -2             | -1             | -2             | 0              | -1             | 75 | 6 | -3             | -2             | -2             | -2             | -2             | -1             |
| 10 | 4 | -2             | -2             | -2             | -1             | -1             | 0              | 43 | 4 | -1             | -1             | -2             | -2             | -1             | 0              | 76 | 6 | -3             | -2             | -2             | -2             | -1             | -2             |
| 11 | 4 | -2             | -2             | -2             | -1             | 0              | -1             | 44 | 4 | -1             | -1             | -2             | -2             | 0              | -1             | 77 | 6 | -2             | -3             | -2             | -2             | -2             | -1             |
| 12 | 2 | -1             | -1             | -1             | 0              | 0              | 0              | 45 | 4 | -2             | -2             | -1             | -1             | -1             | -1             | 78 | 6 | -2             | -3             | -2             | -2             | -1             | -2             |
| 13 | 4 | -2             | -2             | -1             | -2             | -1             | 0              | 46 | 4 | -2             | -1             | -2             | -1             | -1             | -1             | 79 | 6 | -2             | -2             | -3             | -2             | -2             | -1             |
| 14 | 4 | -2             | -2             | -1             | -2             | 0              | -1             | 47 | 4 | -2             | -1             | -1             | -2             | -1             | -1             | 80 | 6 | -2             | -2             | -3             | -2             | -1             | -2             |
| 15 | 2 | -1             | -1             | 0              | -1             | 0              | 0              | 48 | 4 | -1             | -2             | -2             | -1             | -1             | -1             | 81 | 6 | -2             | -2             | -2             | -3             | -2             | -1             |
| 16 | 4 | -2             | -1             | -2             | -2             | -1             | 0              | 49 | 4 | -1             | -2             | -1             | -2             | -1             | -1             | 82 | 6 | -2             | -2             | -2             | -3             | -1             | -2             |
| 17 | 4 | -2             | -1             | -2             | -2             | 0              | -1             | 50 | 4 | -1             | -1             | -2             | -2             | -1             | -1             | 83 | 6 | -3             | -3             | -2             | -2             | -2             | -1             |
| 18 | 2 | -1             | 0              | -1             | -1             | 0              | 0              | 51 | 5 | -2             | -2             | -2             | -1             | -1             | -1             | 84 | 6 | -3             | -3             | -2             | -2             | -1             | -2             |
| 19 | 4 | -1             | -2             | -2             | -2             | -1             | 0              | 52 | 5 | -2             | -2             | -1             | -2             | -1             | -1             | 85 | 6 | -3             | -2             | -3             | -2             | -2             | -1             |
| 20 | 4 | -1             | -2             | -2             | -2             | 0              | -1             | 53 | 5 | -2             | -1             | -2             | -2             | -1             | -1             | 86 | 6 | -3             | -2             | -3             | -2             | -1             | -2             |
| 21 | 2 | 0              | -1             | -1             | -1             | 0              | 0              | 54 | 5 | -1             | -2             | -2             | -2             | -1             | -1             | 87 | 6 | -3             | -2             | -2             | -3             | -2             | -1             |
| 22 | 3 | -1             | -1             | -1             | -1             | 0              | 0              | 55 | 4 | -2             | -2             | -2             | -1             | -1             | -1             | 88 | 6 | -3             | -2             | -2             | -3             | -1             | -2             |
| 23 | 3 | -1             | -1             | -1             | -1             | -1             | 0              | 56 | 4 | -2             | -2             | -1             | -2             | -1             | -1             | 89 | 6 | -2             | -3             | -3             | -2             | -2             | -1             |
| 24 | 3 | -1             | -1             | -1             | -1             | 0              | -1             | 57 | 4 | -2             | -1             | -2             | -2             | -1             | -1             | 90 | 6 | -2             | -3             | -3             | -2             | -1             | -2             |
| 25 | 3 | -2             | -1             | -1             | -1             | -1             | 0              | 58 | 4 | -1             | -2             | -2             | -2             | -1             | -1             | 91 | 6 | -2             | -3             | -2             | -3             | -2             | -1             |
| 26 | 3 | -2             | -1             | -1             | -1             | 0              | -1             | 59 | 5 | -2             | -2             | -2             | -2             | -2             | 0              | 92 | 6 | -2             | -3             | -2             | -3             | -1             | -2             |
| 27 | 3 | -1             | -2             | -1             | -1             | -1             | 0              | 60 | 5 | -2             | -2             | -2             | -2             | 0              | -2             | 93 | 6 | -2             | -2             | -3             | -3             | -2             | -1             |
| 28 | 3 | -1             | -2             | -1             | -1             | 0              | -1             | 61 | 5 | -2             | -2             | -2             | -2             | -2             | -1             | 94 | 6 | -2             | -2             | -3             | -3             | -1             | -2             |
| 29 | 3 | -1             | -1             | -2             | -1             | -1             | 0              | 62 | 5 | -2             | -2             | -2             | -2             | -1             | -2             | 95 | 7 | -3             | -3             | -3             | -2             | -2             | -2             |
| 30 | 3 | -1             | -1             | -2             | -1             | 0              | -1             | 63 | 5 | -3             | -2             | -2             | -1             | -1             | -1             | 96 | 7 | -3             | -3             | -2             | -3             | -2             | -2             |
| 31 | 3 | -1             | -1             | -1             | -2             | -1             | 0              | 64 | 5 | -3             | -2             | -1             | -2             | -1             | -1             | 97 | 7 | -3             | -2             | -3             | -3             | -2             | -2             |
| 32 | 3 | -1             | -1             | -1             | -2             | 0              | -1             | 65 | 5 | -3             | -1             | -2             | -2             | -1             | -1             | 98 | 7 | -2             | -3             | -3             | -3             | -2             | -2             |
| 33 | 4 | -2             | -2             | -1             | -1             | -1             | 0              | 66 | 5 | -2             | -3             | -2             | -1             | -1             | -1             | 99 | 7 | -3             | -3             | -3             | -3             | -2             | -2             |

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