CO 220 Homework #6

Six questions due on Friday, April 3rd.

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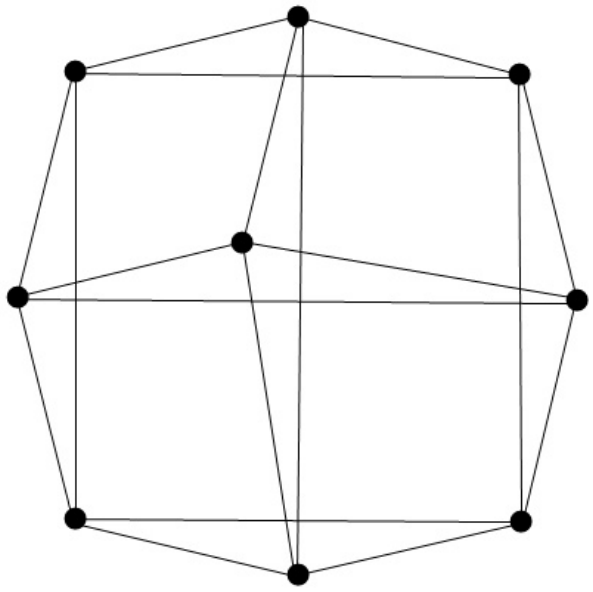
1. Is the graph shown here planar or non-planar? Explain your answer.



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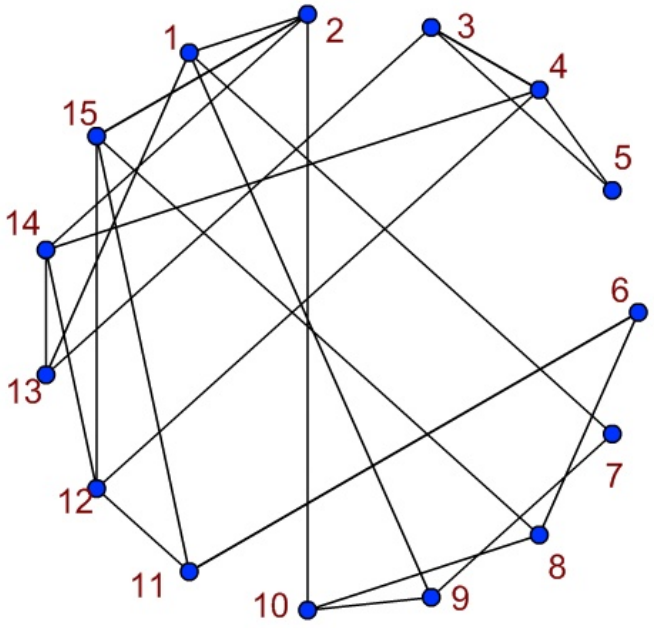
2. Is the graph shown here planar or non-planar? Explain your answer.



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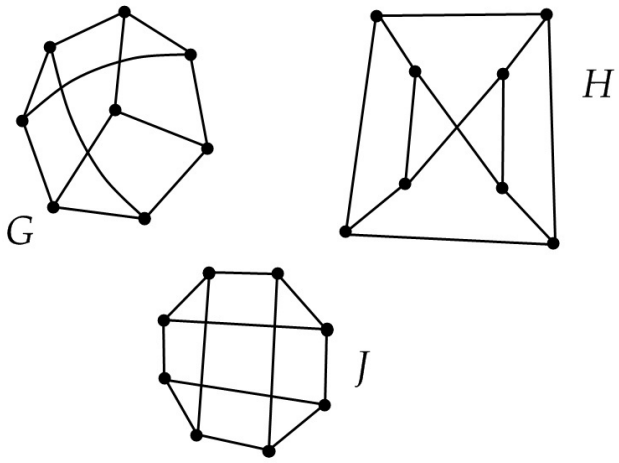
3. Consider the three graphs G, H, and J pictured here:



(a) Two of these graphs are isomorphic. Determine which two, and exhibit an isomorphism between them.
(b) Give an explanation why the remaining graph is not isomorphic with the other two.

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4. Consider a graph $G = (V, E)$ and two vertices v_1 and v_2 in V. For any vertex $w \in V$, let

$$f_{v_1,v_2}(w) = \min\{\text{dist}_G(v_1, w), \text{dist}_G(v_2, w)\}$$

and let

$$F(v_1, v_2) = \sum_{w \in V} f_{v_1,v_2}(w).$$

Describe an algorithm that takes a graph $G = (V, E)$ as input, and gives as output a pair of vertices v_1, v_2 such that $F(v_1, v_2)$ is as small as possible. (You do not have to prove that your algorithm is correct.)

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5. Let $G = (V, E)$ be the graph whose vertices are the two-element subsets of $\{1, 2, 3, 4, 5\}$, in which vertices v and w are adjacent if and only if the sets v and w are disjoint: $\{v, w\} \in E$ if and only if $v \cap w = \varnothing$.
(a) Draw a picture of G showing that it is isomorphic to the Petersen graph.
(b) Is G bipartite? Explain.
(c) Is G planar? Explain.

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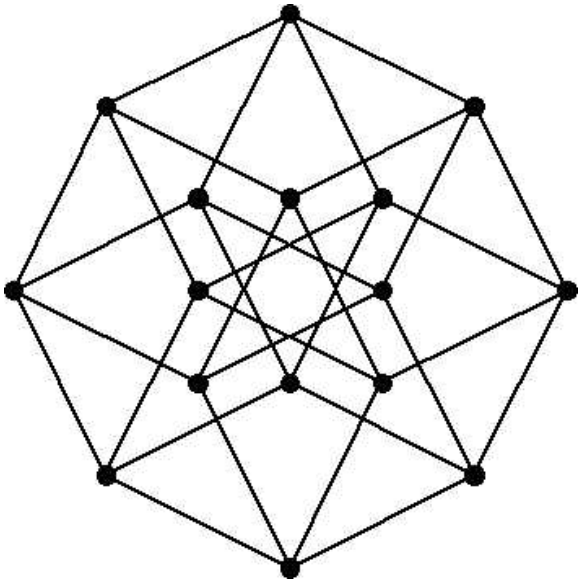
6. Let Q_4 denote the four-dimensional cube graph, shown here:



Show that if e, f, g are *any* three edges of Q_4, then $Q_4 \setminus \{e, f, g\}$ is a non-planar graph.
