CO 220 Homework #5

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due Friday, March 20th.

1(a) What is the maximum number of edges that a (simple) graph with p vertices can have?
1(b) Show that if a (simple) graph G has 17 vertices and 121 edges, then G is connected.

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2. Show that if $G = (V, E)$ is a graph in which every vertex has degree at least 2, then G contains a cycle C as a subgraph.

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3. Let G be a connected graph with $2k \geq 2$ vertices of odd degree. Show that there are k trails W_1, W_2,..., W_k such that each edge of G is in exactly one of these trails.

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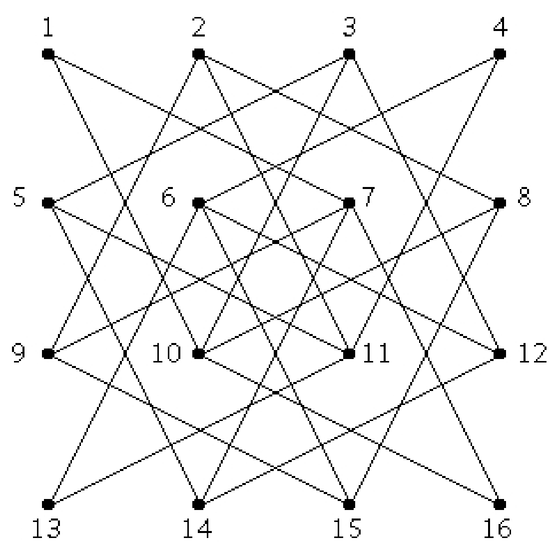
4. The 4–by–4 *Knight's Graph* is shown in the figure below.



Parts (a) to (c) ask for Breadth–First Search trees in this graph with specific root vertices. When growing these trees, at each stage of the algorithm choose the next vertex to be the vertex labelled by the smallest number, among all the available vertices.
(a) Grow a BFS tree rooted at vertex 1.
(b) Grow a BFS tree rooted at vertex 2.

(c) Grow a BFS tree rooted at vertex 6.
(d) What is the maximum distance between any two vertices in this graph? Explain.

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5. For a graph G and integer $k \geq 0$, let n_k be the number of vertices of G that have degree k.
(a) Show that if G is a tree with $p \geq 2$ vertices then $n_0 = 0$ and

$$n_1 = 2 + n_3 + 2n_4 + 3n_5 + \cdots .$$

(b) Consider a graph G with n_k given by the following table:

k	0	1	2	3	4	5
n_k	0	20	8	4	1	2

and $n_k = 0$ for all $k \geq 6$. Assume that G does not contain any cycles. How many connected components does G have?

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6. Is the graph shown below planar or nonplanar? Explain your answer.