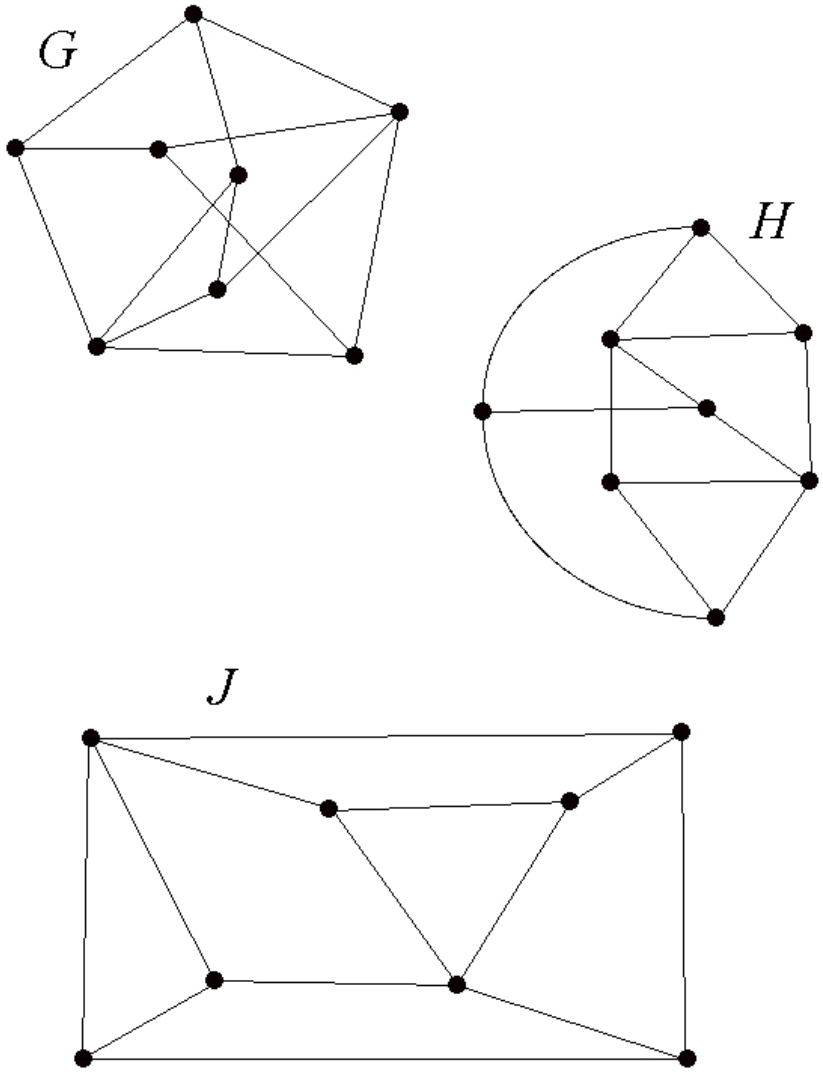
CO 220 Winter 2009 Homework #4.

Five questions due Friday, March 6th, 2009.

1. Consider the three graphs G, H, and J pictured below.



(a) Two of these graphs are isomorphic. Determine which two, and exhibit

an isomorphism between them.
(b) Give an explanation why the remaining graph is not isomorphic with the other two.

2.
(a) Show that a 4-regular graph can not contain a cut-edge.
(b) Give an example of a 3-regular graph that does contain a cut-edge.

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3. Let G be a graph in which every vertex has degree at least k.

(a) Show that G contains a path with at least k edges.
(Hint: consider a longest path P in G.)

(b) Show that G contains a cycle with at least $k+1$ edges.
(Hint: consider the edges incident at one of the ends of P.)

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4. Let G be a graph with p vertices, in which every vertex has degree at least $p/5$. Show that G has at most 4 connected components.

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5. Let G be a connected graph in which the number of edges equals the number of vertices. Show that G contains **exactly one** cycle as a subgraph.
