CO 220 Homework Assignment #3

Due Friday, February 13th, 2009.

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1. How many ways are there to roll a total of 25 on five six-sided dice? (Show your calculation.)

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2. Let a_n be the number of compositions of size n in which each part is at least three.

(a) Show that

$$A(x) = \sum_{n=0}^{\infty} a_n x^n = \frac{1-x}{1-x-x^3}.$$

(b) Show that $a_0 = 1$, $a_1 = 0$, $a_2 = 0$, and for all $n \geq 3$, $a_n = a_{n-1} + a_{n-3}$.

(c) Compute a_n for all $0 \leq n \leq 16$.

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3. Let b_n be the number of compositions of size n in which each part is at least two, and which have an odd number of parts.

(a) Show that

$$B(x) = \sum_{n=0}^{\infty} b_n x^n = \frac{x^2 - x^3}{1 - 2x + x^2 - x^4}.$$

(b) Derive initial conditions and a recurrence relation that determines the sequence (b_n) for all $n \geq 0$.

(c) Compute b_n for all $0 \leq n \leq 12$.

(over...)

4. Consider the power series

$$C(x) = \sum_{n=0}^{\infty} c_n x^n = \frac{1 - 4x + 5x^2}{1 - 4x + 5x^2 - 2x^3} = 1 + 2x^3 + 8x^4 + 22x^5 + 52x^6 + \cdots .$$

(a) Derive initial conditions and a recurrence relation that determines the sequence (c_n) for all $n \geq 0$. (You don't need to compute the sequence.)

(b) Use Partial Fractions to obtain a formula for the coefficient c_n as a function of n. (Hint: $1 - 4x + 5x^2 - 2x^3 = (1 - 2x)(1 - 2x + x^2)$.)

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5. Consider the power series

$$G(x) = \sum_{n=0}^{\infty} g_n x^n = \frac{x + 7x^2}{1 - 3x^2 - 2x^3} = x + 7x^2 + 3x^3 + 23x^4 + 23x^5 + 75x^6 + \cdots .$$

Use Partial Fractions to obtain a formula for the coefficient g_n as a function of n.