CO 220 Homework Assignment #2

Due Friday, January 30th, 2009.

1. In this question one is dealt a hand of 5 cards from a standard deck of 52 cards (4 suits, Ace to King), which we call a *poker hand*. Briefly explain your answer to each part.

(a) How many poker hands are there in total?

(b) How many poker hands have one pair (but no better)?

(c) How many poker hands have two pairs (but no better)?

(d) How many poker hands have three of a kind (but no better)?

(e) How many poker hands are straight flushes?

2. In *Draw Poker* one is dealt a poker hand and then has an opportunity to exchange some (or none, or all) of the cards. Suppose you are dealt the hand 8♠ 8♢ 8♣ $A\heartsuit$ 5♢.

(a) If you throw away the 5♢ and draw one card, what is the probability that your hand improves (to Full House or Quads)?

(b) If you throw away both $A\heartsuit$ 5♢ and draw two cards, what is the probability that your hand improves (to Full House or Quads)?

3. Prove that for any integers $n \geq 1$ and $t \geq 2$:

$$\binom{n+t-1}{t-1} = \sum_{k=0}^{n} \binom{n-k+t-2}{t-2}.$$

4. For each $t \geq 1$, let $f(t)$ denote the probability that a randomly chosen $(2t)$-element multiset with elements of t types is such that every type of element occurs at least once.

(a) Give a formula for $f(t)$ as a function of t.

(b) Compute $f(t)$ to 6 decimal places for each $1 \leq t \leq 6$.

5. Consider a randomly chosen permutation σ of the set $\{a, b, c, d, e, f, g, h\}$.

(a) What is the probability that in σ the letters a and b are **not** adjacent? In other words, what is the probability that σ does not contain either ab or ba as a substring?

(b) What the probability that in σ the letters a and b are not adjacent, and the letters b and c are not adjacent?