CO 220 Winter 2009 Homework #1

Four questions due Friday, January 16th.

---

1. There are 6^4 possible outcomes when rolling four ordinary 6-sided dice. Count how many of these outcomes are of each of the following types:
(a) no pairs;
(b) exactly one pair;
(c) exactly two pairs;
(d) three of a kind (but not four);
(e) four of a kind.
(As a check, notice that every outcome is of exactly one of these types, so the answers must sum up to 6^4.)

---

2. Continuing Exercise 1, say that when rolling four dice an outcome is a *run* if it shows four different but consecutive numbers. That is, the dice show $\{1, 2, 3, 4\}$ or $\{2, 3, 4, 5\}$ or $\{3, 4, 5, 6\}$ in some order. Is rolling a run *more* or *less* likely than rolling three (but not four) of a kind? Explain.

(over. . .)

---

3. Assume that people's birthdays are independently and uniformly distributed over the seven days of the week from Sunday to Saturday. (This is a reasonable approximation since twins are relatively rare.) Let $p(n)$ denote the probability that in a randomly chosen group of n people, at least two of them are born on the same day of the week.
(a) Give a formula for $p(n)$ as a function of n. (Check that your answer gives $p(2) = 1/7$.)
(b) Complete the following table, expressing your answers both as exact fractions and as approximations to six decimal places.

n	$p(n)$	$p(n)$
1	0/1	0.000000
2	1/7	0.142857
3		
4		
5		
6		
7		
$n \geq 8$	1/1	1.000000

4(a) Show that for $n, t, k \geq 1$, the number of n–element multisets with elements of t types in which **exactly** k different types of element occur (at least once each) is $\binom{t}{k}\binom{n-1}{k-1}$.

4(b) Show that for $n, t \geq 1$,

$$\binom{n+t-1}{t-1} = \sum_{k=1}^{t} \binom{t}{k}\binom{n-1}{k-1}.$$
