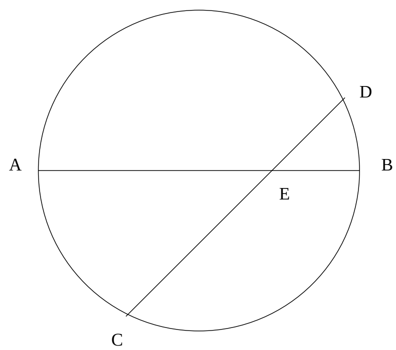
SPECIAL K
Saturday 27 October 2001
9 a.m. to 12 noon

1. Prove that
$$\sum \frac{1}{i_1 i_2 \cdots i_k} = 2001$$
where the summation is over all non-empty subsets $\{i_1, i_2, \ldots, i_k\}$ of the set $\{1, 2, \ldots, 2001\}$.

2. In the diagram, a diameter AB of a circle intersects a chord CD at the point E. If $CE = 7$, $DE = 1$ and $\angle BED = \frac{\pi}{4}$, determine the radius of the circle.



3. For any positive integer k consider the sequence
$$a_n = \sqrt{k + \sqrt{k + \cdots + \sqrt{k}}} \qquad n \text{ square} - \text{root signs}$$
   (a) Show that the sequence $\{a_n\}$ converges for every fixed integer $k \geq 1$.
   (b) Find all integers k so that the limit is an integer.
   (c) Show that if k is odd, then the limit is irrational.

4. Determine all functions $f : \mathbb{R}^+ \to \mathbb{R}$ such that
$$f(x + y) = f(x^2 + y^2)$$
for all $x, y \in \mathbb{R}^+$, where $\mathbb{R}^+$ is the set of all strictly positive real numbers.

5. Find all pairs (m, n) of positive integers so that
$$GCD((n+1)^m - n,\ (n+1)^{m+3} - n) > 1.$$

BIG E
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9 a.m. to 12 noon

1. Find all pairs of non-negative integers x and y such that

$$x - y = x^2 + xy + y^2.$$

2. Evaluate

$$\int_0^\pi \log(\sin x) dx.$$

3. We are given $n \geq 4$ points in the plane such that the distance between any two of them is an integer. Prove that at least

$$\frac{\binom{n}{2}}{6}$$

   of these distances are divisible by 3.

4. Evaluate

$$\sum_{k \in S} \left\lfloor \sqrt{\frac{n}{k}} \right\rfloor,$$

   where

$$S = \{k \in \mathbb{N} \ : \ a \in \mathbb{N},\ a^2 \,|\, k \Rightarrow a = 1\}.$$

5. Let $L : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation on $\mathbb{R}^n$, where n is an integer greater than one. Prove that there exists a two-dimensional subspace $V \subseteq \mathbb{R}^n$ such that $L(V) \subseteq V$.