Algebraic Aspects of t-Intersecting Families

Open Problems in Algebraic Combinatorics Workshop

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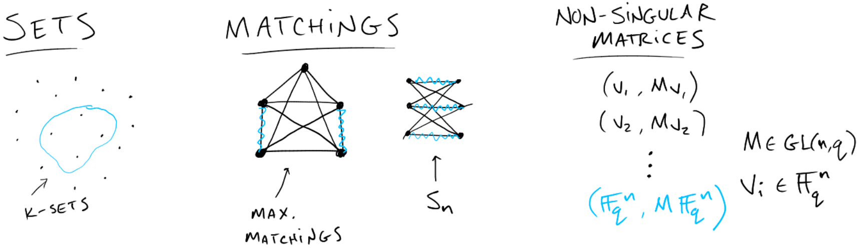
t-intersection

Throughout this talk, we assume that

- $t \in \mathbb{N}$,
- $\mathcal{X} = \{\mathcal{X}_n\}_{n=0}^{\infty}$ is a poset graded by inclusion,
- and objects $X \in \mathcal{X}_n$ are composed of atomic elements:



so there is some natural notion of t-intersection.

t-EKR Theorems

The generic t-EKR theorem:
If $\mathcal{F} \subseteq \mathcal{X}_n$ is t-intersecting, then $|\mathcal{F}| \leq |\mathcal{X}_{n-t}|$ for n suff. large, and the canonically t-intersecting families attain this bound:

$$\mathcal{F}_x := \{X \in \mathcal{X}_n : x \subseteq X\} \text{ for some } x \text{ s.t. } |x| = t.$$

Is there a general way of proving such results?

The Ratio Bound

Take $\Gamma_t = (\mathcal{X}_n, E)$ such that $X \sim X'$ if X, X' do not t-intersect.

Theorem (Delsarte, Hoffman)

Let $\widetilde{A}(\Gamma_t)$ be a pseudo-adjacency matrix of a regular N-vertex graph Γ_t with eigenvalues $\{\theta_i\}$ and a corresponding system of orthonormal eigenvectors $\{v_i\}$. If S is an independent set of Γ_t, then

$$\frac{|S|}{N} \leq \frac{-\theta_{\min}}{\theta_{\max} - \theta_{\min}}.$$

If equality holds, then

$$1_S \in Span\{v_{\max}\} \oplus Span\{v_i : \theta_i = \theta_{\min}\}.$$

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If equality holds, also $\alpha(\Gamma_t) =$ Shannon cap. of $\Gamma_t = \vartheta(\Gamma_t)$.

Motivation

Too many domains $\mathcal{X}$ to consider:

1. Multislices
2. Hypermatchings
3. Injections $[k] \hookrightarrow [n]$
4. Gelfand pairs of the form $(GL(n,q), H(n,q))$
5. $\vdots$

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Want to show Γ_t is ratio tight, but with little calculation.

Need a unifying framework.

Hecke Algebras/H.C.C.'s

Each of these domains $\mathcal{X}$ is a *homogeneous space*, i.e.,

$$\mathcal{X} = \{\mathcal{X}_n\}_{n=0}^{\infty} \quad \text{such that } \mathcal{X}_n \cong G_n/H_n$$

for some family $(G_i, H_i)_{i=0}^{\infty}$.

Let $\mathbb{C}\mathcal{X}_n$ be the space of complex-valued functions on $\mathcal{X}_n$. Then

$$\mathrm{End}_{G_n}\mathbb{C}\mathcal{X}_n \cong \mathbb{C}[H_n \backslash G_n / H_n].$$

The *orbitals* of $\mathcal{X}_n$ are the orbits of diagonal action $G_n \curvearrowright \mathcal{X}_n \times \mathcal{X}_n$.

The orbitals are a canonical basis of the endomorphism algebra:

$$\mathrm{End}_{G_n}(\mathcal{X}_n) \cong \bigoplus_{i \in \mathbb{I}(\mathcal{X}_n)} \mathrm{Mat}_{m_i, m_i}(\mathbb{C}).$$

Note that $\mathrm{End}_{G_n}(\mathcal{X}_n)$ is commutative if and only if $m_i = 1$ for all i.

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What conditions on $\mathrm{End}_{G_n}\mathbb{C}\mathcal{X}_n$ *guarantee* Γ_t *is ratio-tight?*

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- stability/polynomiality of structure constants ($\cap$ numbers).

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"It claims to be fully automatic, but actually you have to push this little button here."

-Gentleman John Killian

Representation Theory

In general, G_n-irreducibles do not have "names", but there are some notable exceptions.

- Coxeter groups (partitions)
- Finite groups of Lie type (partition-valued functions)
- Abelian groups (group elements)

We require our irreducibles to have "names".

Representation Stability

When irreps have names, we can talk about *representation stability*.

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When irreps have names, we can talk about *representation stability*.
Ex. $V_{2,n} \cong S_n \curvearrowright$ 2-matchings of K_n.

$$\begin{aligned}
V_{2,4} &= [4] \oplus [2,2] \\
V_{2,5} &= [5] \oplus [4,1] \oplus [3,2] \oplus [2,2,1] \\
V_{2,6} &= [6] \oplus [5,1] \oplus [4,2]^2 \oplus [3,2,1] \oplus [2,2,2] \\
V_{2,7} &= [7] \oplus [6,1] \oplus [5,2]^2 \oplus [4,3] \oplus [4,2,1] \oplus [3,2,2] \\
V_{2,8} &= [8] \oplus [7,1] \oplus [6,2]^2 \oplus [5,3] \oplus [5,2,1] \oplus [4,4] \oplus [4,2,2] \\
V_{2,9} &= [9] \oplus [8,1] \oplus [7,2]^2 \oplus [6,3] \oplus [6,2,1] \oplus [5,4] \oplus [5,2,2] \\
&\vdots \\
V_{2,n} &= [n] \oplus [n-1,1] \oplus [n-2,2]^2 \oplus [n-3,3] \\
&\qquad \oplus [n-3,2,1] \oplus [n-4,4] \oplus [n-4,2,2] \\
&= \emptyset_n \oplus [1]_n \oplus [2]_n^2 \oplus [3]_n \oplus [2,1]_n \oplus [4]_n \oplus [2,2]_n
\end{aligned}$$

where $[\lambda]_n := (n - |\lambda|, \lambda_1, \cdots, \lambda_\ell)$ provided $n \geq |\lambda| + \lambda_1$.

Let $c_i(\sigma) := \#$ i-cycles of $\sigma \in S_n$.

Theorem (Frobenius 1904)

For each λ, there is a unique polynomial $P_\lambda \in \mathbb{Q}[c_1, \cdots, c_n]$ of degree $|\lambda|$ such that

$$\chi^{[\lambda]_n}(\sigma) = P_\lambda(\sigma) \quad \text{for all } n \geq |\lambda| + \lambda_1 \text{ and } \sigma \in S_n.$$

We call these *character polynomials*.

Corollary

Let $\lambda \vdash t$. Then $\dim[\lambda]_n = poly(n, t)$.

Representation Stability in EKR

Let C be the $\mathcal{X}_n \times \mathcal{X}_{n,t}$ matrix whose columns are the canonically t-intersecting families, i.e.,

$$C_{X,x} = \begin{cases} 1 & \text{if } x \subseteq X; \\ 0 & \text{otherwise.} \end{cases}$$

The map $C : \mathbb{C}\mathcal{X}_{n,t} \to \mathbb{C}\mathcal{X}_n$ is G_n-equivariant.

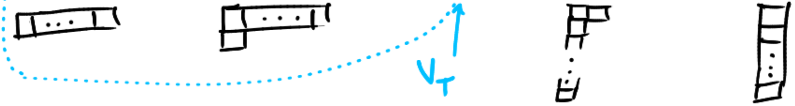
By Schur's lemma, for any $V \in \mathbb{I}(\mathcal{X}_{n,t})$ the restriction $C|_V$ is the zero map or an isomorphism.

$$\text{Span}\{\text{canonically t-intersecting families}\} \subseteq \bigoplus_{\lambda \leq \mathsf{T}} V_\lambda \subset \mathbb{C}\mathcal{X}_n$$

where V_T is the "greatest" irreducible of $\mathbb{I}(\mathcal{X}_{n,t})$.

Ex. Symmetric Group (Rev-Lex total order)

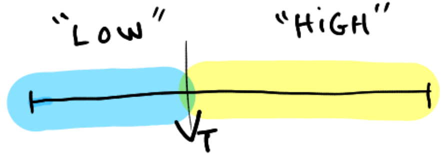
$$[n] < [n-1,1] < \cdots < [2,1^{n-2}] < [1^n]$$



Ex. $\mathbb{Z}_2^n$ ("Hamming weight" partial order)

$$\emptyset < \{i\} < \cdots < \overline{\{i\}} < \{1,2,\cdots,n\}$$

$\Pi(X_n)$



Low frequencies vs. High frequencies

Quick recap:

- $\mathcal{X}_n \cong S_n/H_n$,
- eigenspaces of $\widetilde{A} \in \mathrm{End}_{S_n}\mathbb{C}\mathcal{X}_n$ are sums of S_n-irreducibles,
- via, say, representation stability, the dimensions of the low frequencies are low-degree polynomials, i.e., $O(poly(n, t))$.

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Vanishingly few S_n-irreducibles $[\lambda] \in \mathbb{I}(\mathcal{X}_n)$ are less than V_T.

Fortunately, the high frequencies have high dimension!

Theorem (Ellis, Friedgut, Pilpel '11)

Let $t \in \mathbb{N}$. If $\lambda_1 < n - t$ and $(\lambda')_1 < n - t$, then $\dim[\lambda] = \Omega(n^{t+1})$.

Open Question: Prove a q-analogue of this for $GL(n,q)$.

An Eigenvalue Bound for Association Schemes

Lemma

Let θ_i be the eigenvalue corresponding to the irrep V_i of the associate A_Ω with valency $|\Omega|$. Then

$$|\theta_i| \leq \sqrt{\frac{|\mathcal{X}_n||\Omega|}{\dim V_i}}.$$

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Proof.

Let $\omega^i \in \mathbb{C}\mathcal{X}_n$ be the ith spherical function, so that $\theta_i = \langle \omega^i, 1_\Omega \rangle$. For any spherical function ω^i, we have $\langle \omega^i, \omega^i \rangle = \frac{|\mathcal{X}_n|}{\dim V_i}$. By Cauchy-Schwarz, we have

$$|\theta_i| = |\langle \omega^i, 1_\Omega \rangle| \leq \sqrt{\langle \omega^i, \omega^i \rangle \langle 1_\Omega, 1_\Omega \rangle} \leq \sqrt{\frac{|\mathcal{X}_n||\Omega|}{\dim V_i}}.$$

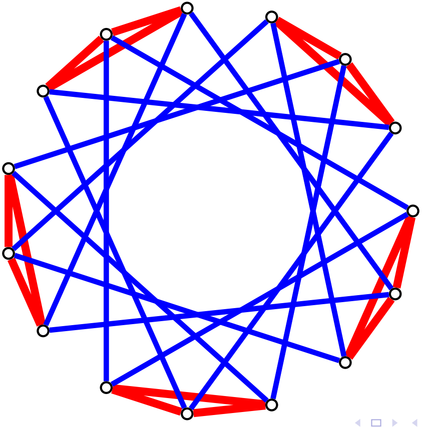
□

Orbitals

We also require that our orbitals have "names".

Ideally, we want orbitals to have same "names" as the irreps (duality).

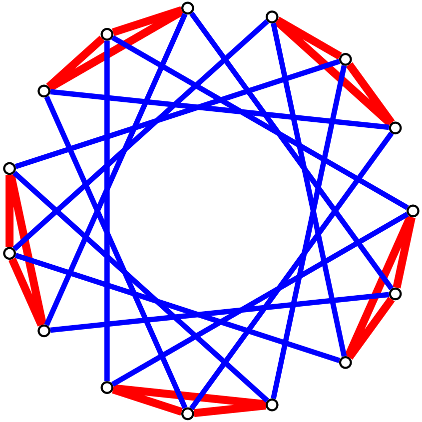
The "names" of the orbitals are the *colored isomorphism types* ρ.
Ex. $G_5 = S_{15}$ and $H_5 = (S_3 \wr S_5)$



Valencies of Orbitals

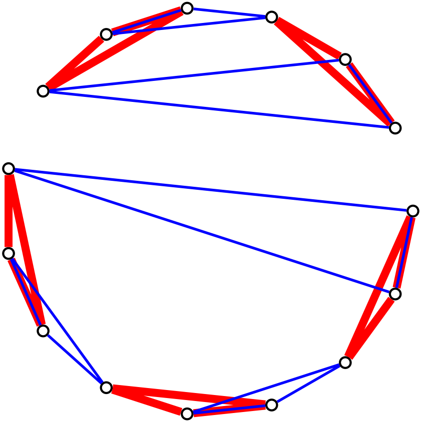
Let $|\Omega_\rho|$ be the valency of orbital ρ. For $\sigma_\rho \in S_n$, we have

$$|H_n \sigma_\rho H_n| = \frac{|H_n|^2}{|H_n \cap \sigma_\rho^{-1} H_n \sigma_\rho|} \quad \text{and} \quad |\Omega_\rho| = \frac{|H_n|}{\left| \underbrace{\color{red}{H_n} \cap \color{blue}{\sigma_\rho^{-1} H_n \sigma_\rho}}_{\text{“}color-preserving\ \ auts\text{”}} \right|}.$$



Orbitals

Connected Components



$$3(3,2) = (9,6) \vdash 15$$

Orbitals

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Want the probability of $X^* \cup X$ being connected as large as possible!

- Symmetric Group S_n:

$$\mathbb{P}_{\sigma,\pi}[\sigma^{-1}\pi \text{ is a } n\text{-cycle}] = \frac{1}{n}.$$

- Perfect Matchings of K_{2n}:

$$\mathbb{P}_{m,m'}[m \cup m' \text{ is a } 2n\text{-cycle}] \approx \frac{1}{\sqrt{n}}.$$

A Linear System of Equations for the Low Frequencies

Recall we want a $\widetilde{A}(\Gamma_t) \in \mathrm{End}_{G_n}\mathbb{C}\mathcal{X}_n$ such that

$$\frac{|\mathcal{X}_{n-t}|}{|\mathcal{X}_n|} = \frac{-\theta_{\min}}{\theta_{\max} - \theta_{\min}}.$$

Normalizing gives us

$$\frac{|\mathcal{X}_{n-t}|}{|\mathcal{X}_n|} = \frac{-\theta_{\min}}{1 - \theta_{\min}}.$$

Solve for $\theta_{\min}$ and let $\zeta_{n,t}$ be the solution. Note that

$$|\zeta_{n,t}| \propto \left(\frac{|\mathcal{X}_{n-t}|}{|\mathcal{X}_n|}\right) \propto 1/(n)_t.$$

A Linear System of Equations for the Low Frequencies

The A_λ's are all spanning subgraphs of Γ_t with large valency.

$$\sum_{\lambda < \mathrm{T}} \theta_{triv}(A_\lambda) x_\lambda = 1 \tag{1}$$

$$\sum_{\lambda < \mathrm{T}} \theta_\mu(A_\lambda) x_\lambda = \zeta_{n,t} \quad \forall \text{ non-triv eigenspaces } \mu < \mathrm{T} \tag{2}$$

$$\sum_{\lambda < \mathrm{T}} \theta_\mu(A_\lambda) x_\lambda = \zeta_{n,t} \quad \mu = \mathrm{T} \tag{3}$$

One more equation than there are unknowns!

But if a solution x^* exists, then

$$\widetilde{A}(\Gamma_t) = \sum_{\lambda < \mathrm{T}} x^*_\lambda A_\lambda$$

gives us a pseudo-adjacency matrix with the desired eigenvalues on the low frequencies.

(1) and (2) $\Rightarrow$ (3)

Write a canonically t-intersecting family 1_S in the Fourier basis. By Parseval, the Fourier mass of 1_S is

$$\alpha := |\mathcal{X}_{n-t}|/|\mathcal{X}_n|.$$

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$$0 = 1_S^\top \widetilde{A}(\Gamma_t) 1_S = \sum_{\lambda \leq T} \theta_\lambda \underbrace{W(\lambda)}_{\lambda-mass}.$$

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Plugging in the low frequency eigenvalues on the RHS gives

$$0 = \sum_{\lambda \leq \mathsf{T}} \theta_\lambda \underbrace{W(\lambda)}_{\lambda-mass} = \alpha^2 + \zeta_{n,t}(\alpha - \alpha^2 - W(\mathsf{T})) + \theta_{\mathsf{T}} W(\mathsf{T}).$$

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We have $\zeta_{n,t}(\alpha - \alpha^2) = -\alpha^2$, so it must be that $\theta_\mathsf{T} = \zeta_{n,t}$.

Inequalities for the High Frequencies

We need to be sure that $\zeta_{n,t}$ is in fact the least eigenvalue!

No control over signs of eigenvalues either, so we must show:

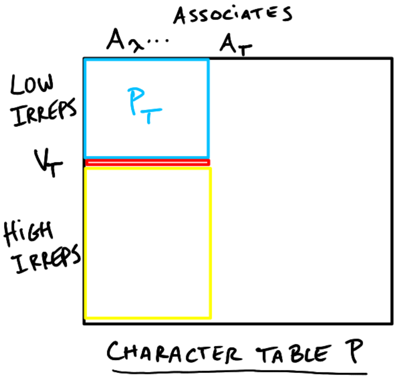
$$\sum_{\boldsymbol{\lambda} < \mathsf{T}} \theta_{triv}(A_{\boldsymbol{\lambda}}) x_{\boldsymbol{\lambda}} = 1 \tag{4}$$

$$\sum_{\boldsymbol{\lambda} < \mathsf{T}} \theta_{\mu}(A_{\boldsymbol{\lambda}}) x_{\boldsymbol{\lambda}} = \zeta_{n,t} \quad \forall \text{ eigenspaces } \mu < \mathsf{T} \tag{5}$$

$$\left| \sum_{\boldsymbol{\lambda} < \mathsf{T}} \theta_{\rho}(A_{\boldsymbol{\lambda}}) x_{\boldsymbol{\lambda}} \right| \leq |\zeta_{n,t}| \quad \forall \text{ eigenspaces } \rho > \mathsf{T} \tag{6}$$

Keep in mind the dimensions of the eigenspaces $> \mathsf{T}$ are $\Omega(n^{t+1})$.

Solving the Linear System



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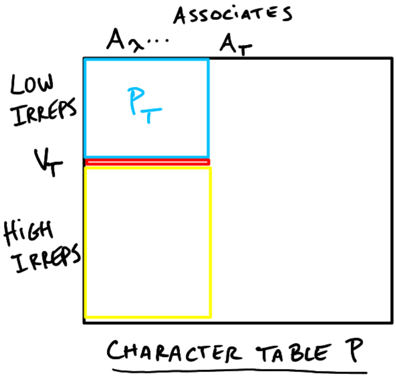

P_T must be invertible!

If $P = LU$, then all leading principal minors of P are nonzero.

Character tables often admit LU-factorizations!

Bounding the High Frequencies

Let x^* be the solution to the linear system. Then

$$\begin{aligned}
|\theta_\rho| &= \left| \sum_{\lambda < T} x^*_\lambda \theta_\rho(A_\lambda) \right| \\
&\leq O(1) \max_\lambda |x^*_\lambda| \max_\lambda |\theta_\rho(A_\lambda)| \\
&\leq O(1) \max_\lambda |x^*_\lambda| \sqrt{\frac{|\mathcal{X}_n||\Omega_\lambda|}{n^{t+1}}} \\
&\leq \frac{O(1)}{|\Omega_\lambda|} \sqrt{\frac{|\mathcal{X}_n||\Omega_\lambda|}{n^{t+1}}} \\
&\leq \sqrt{\frac{O(|\mathcal{X}_n|/|\Omega_\lambda|)}{n^{t+1}}}
\end{aligned}$$

If $O(|\mathcal{X}_n|/|\Omega_\lambda|)$ is small enough, i.e., the probability of $X^* \cup X$ connected is large enough, then since $|\zeta_{n,t}| \propto 1/(n)_t$, we have

$$|\theta_\rho| = o(|\zeta_{n,t}|). \quad \square$$

Open Problems

There's a nascent rep. stability theory for finite groups of Lie type.

- t-EKR for $GL(n, q)$ via $SL(n, q)$? $PGL(n, q)$ via $PSL(n, q)$?
- t-EKR for the Gelfand pair $(GL(2n, q), Sp(2n, q))$, i.e., symplectic forms over $\mathbb{F}_q^{2n}$?

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Orbital valencies seem to have the right asymptotics.

Is there a rev-lex-like ordering of the conjugacy-classes and irreps of the $SL(n,q)$ character table C such that $C = LU$?

Is commutativity of the Hecke algebra needed?

- t-EKR for $\mathcal{X}$ with a non-commutative Hecke algebra?

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Can we beat Cauchy-Schwartz for the eigenvalue bound? (weak!)

That's all. Thanks!