# Erdős-Ko-Rado Theorems for Groups - Open Problems

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## Open Question

For which groups $G$ and subgroups $H \leq G$ does

$$\bigcup_{x \in G} xHx^{-1}$$

contain a subgroup, other than $H$?

## Open Question

Is there a group property that ensures such subgroups exist or don't exist?

## Open Question

If $G \times H$ is the **internal** direct product, then $\Gamma_{(G \times H)} = \Gamma_G \times \Gamma_H$.

Are there other group products that are interesting?

## Open Question

For which groups $G$ does $\mathrm{der}(G)$ generate the group?

This is a hard one! See:
*Groups generated by derangements*
R. A. Bailey, Peter J. Cameron, Michael Giudici and Gordon F. Royle

## Open Question

Find and prove the interesting patterns in the eigenvalues for $\Gamma_{\mathrm{Sym}(n)}$.


1. C. Y. Ku, D. B. Wales Eigenvalues of the derangement graph, J.Combin. Theory Ser. A 117 (2010) 289?312.
2. Solving the Ku?Wales conjecture on the eigenvalues of the derangement graph Cheng Yeaw Ku, Kok Bin Wong.

## Open Question

Are there other groups that have so many cliques of size $n$ that we can prove the group that the EKR module property using the cliques?

## Open Question

Is there a group property that would imply that there are enough big cliques for the group to have the EKR module property?


- Jun Wang and Sophia J. Zhang An Erdős-Ko-Rado-type theorem in Coxeter groups
- Chris Godsil and Karen Meagher. A new proof of the Erdős-Ko-Rado theorem for intersecting families of permutations. European Journal of Combinatorics, 30(2):404-414, 2009.

## Open Question

Can we use the graph homomorphism argument to bound the size cocliques in derangements graph (other than when we use $\Gamma_H \to \Gamma_G$ for $H \leq G$.)

## Open Question

For which 2-transitive groups does the character $\chi(g) = \mathrm{fix}(g) - 1$ give the least eigenvalue of the derangement graph?

## Open Question

What's up with the 1-transitive groups that have $\frac{-|\mathrm{der}(G)|}{n-1}$ as a least eigenvalue?

## Open Question

How can the ratio bound be sharpened?

## Open Question

For any 2-transitive group does there exist a weighting for the derangement graph so that equality holds in the ratio bound?

## Open Question

Let $G$ be a 1-transitive group. When is it possible to weight the conjugacy classes so that the ratio bound holds with equality?

## Open Question

What are some interesting 1-transitive group for which it is not possible to weight the conjugacy classes so that the ratio bound holds with equality?

## Open Question

What interesting families of groups have EKR property?

## Open Question

Which 2-transitive groups have the strict EKR property?

## Open Question

For 2-transitive groups (or 1-transitive groups!), what are the boolean vectors in the span of vectors $v_{i,j}$?

## Open Question

Which 1-transitive groups have interesting intersecting set of permutations?

## Open Question

In a transitive group what is the largest set of intersecting permutations that is closed under taking conjugation?

## Open Question

In a 2-transitive group are all maximum cocliques of the derangement graph either groups or cosets of groups?

## Open Question

In a 2-transitive group, can there be a subgroup that is also a maximum coclique that is not isomorphic to the stabilizer of a point?

## Open Question

When does a group have non-conjugate subgroups that give the same induced representation?