Search on the Complete Graph by Lackadaisical Quantum Walk

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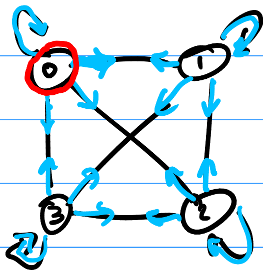
First, let's review search with one self-loop per vertex, which Harmony covered.

K_4



Initial state

$$|\psi(0)\rangle = \frac{1}{\sqrt{16}}|0,0\rangle + \frac{1}{\sqrt{16}}|0,1\rangle + \dots + \frac{1}{\sqrt{16}}|3,3\rangle$$

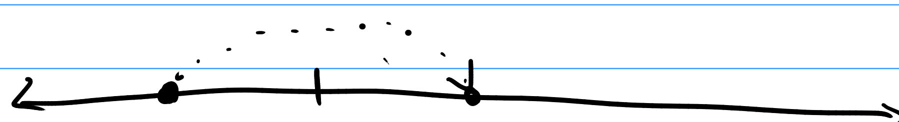
$U = SCQ$ Oracle query

$$Q|\psi(0)\rangle = \frac{-1}{\sqrt{16}}\left(|0,0\rangle + |0,1\rangle + |0,2\rangle + |0,3\rangle\right)$$

$$+ \frac{1}{\sqrt{16}}\left(|1,0\rangle + \dots + |3,3\rangle\right)$$

Grover Coin

at vertex 1: $\mu = \dfrac{\frac{-1}{\sqrt{16}} + \frac{-1}{\sqrt{16}} + \frac{-1}{\sqrt{16}} + \frac{-1}{\sqrt{16}}}{4} = \dfrac{-1}{\sqrt{16}}$

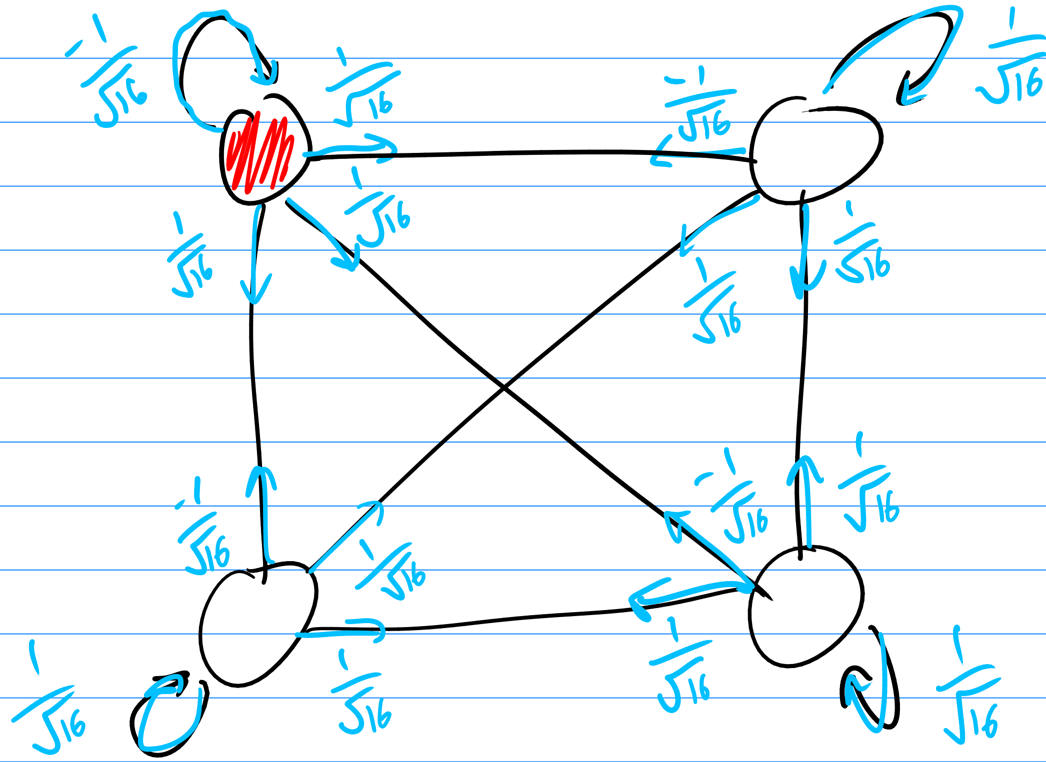


$\frac{-1}{\sqrt{16}}$

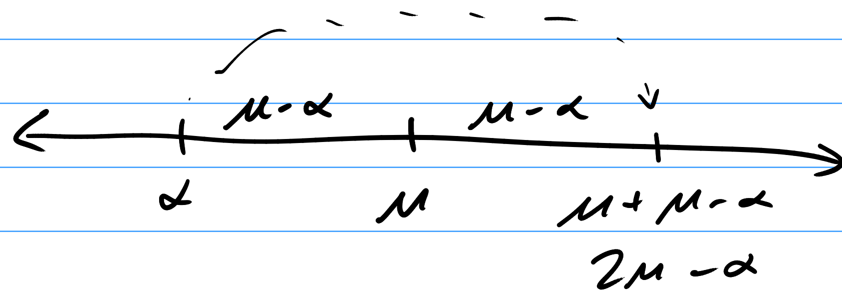
$$CQ|\psi(0)\rangle = \frac{-1}{\sqrt{16}}\left(|0,0\rangle + |0,1\rangle + |0,2\rangle + |0,3\rangle\right)$$

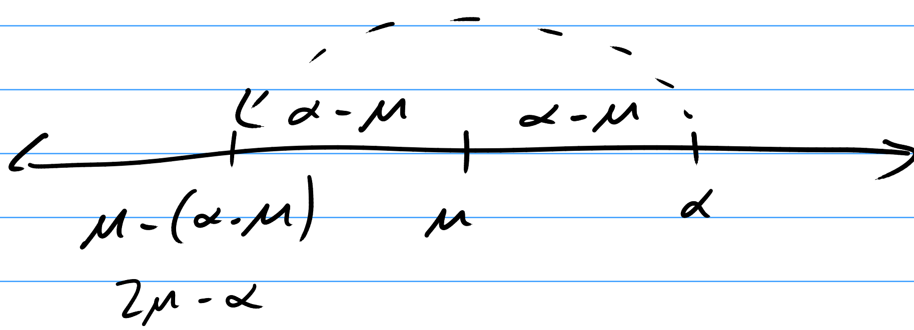
$$+ \frac{1}{\sqrt{16}}\left(|1,0\rangle + \dots + |3,3\rangle\right)$$

Flip-flop shift



Coin





$$\alpha \to 2\mu - \alpha$$

If we repeat this, the probability at the marked vertex reaches 1.
See noloops.ipynb.

This was discovered by Ambainis, Kempe, Rivosh (2005)

What if we have...

No self-loops? Prob reaches $\frac{1}{2}$. See oneloop.ipynb

Multiple self loops? Lackadaisical quantum walk. See multipleloops.ipynb

N=4

$|0,0\rangle, |0,1\rangle, |0,2\rangle, |0,3\rangle, |0,4\rangle, |0,5\rangle$

$|1,0\rangle, |1,1\rangle, |1,2\rangle, |1,3\rangle, |1,4\rangle, |1,5\rangle$

No loops: prob $\frac{1}{2}$

1 loop: " 1

2 loops: " 0.9

3 loops: " 0.75

↓ ↓

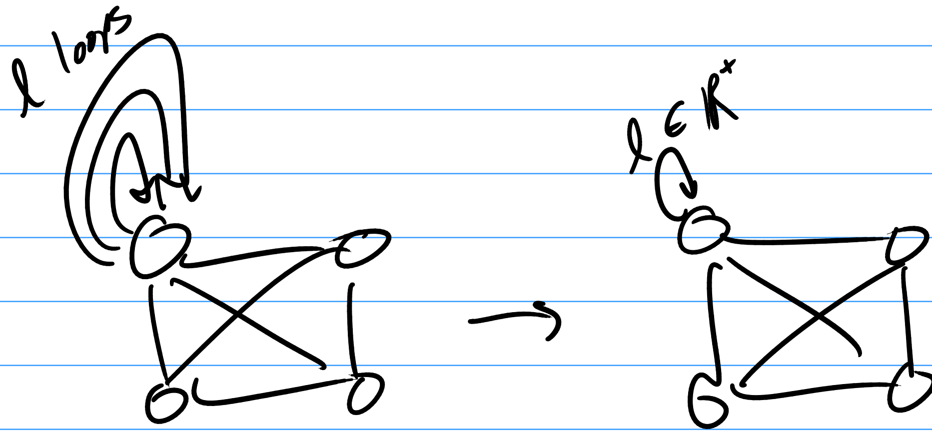
worse

Main point: There is some optimal amount of laziness.

This is from "Grover search by lackadaisical quantum walk" by Wong (2015)

Some additional results:

We can replace multiple self-loops with a single weighted self-loop:



This is "Coined quantum walks on weighted graphs" by Wong (2017)

We can explore search on other graphs

2D torus, "Faster Search by Lackadaisical Quantum Walk," Wong (2017)

2D torus with multiple marked vertices, Nahimovs (2019)

2D torus with multiple marked vertices, 1D cycle, Hanoi network, Giri & Korepin (2020)

Complete bipartite graphs, "Search by Lackadaisical Quantum Walk with Nonhomogeneous Weights, " Rhodes and Wong (2019)

Vertex-transitive graphs, higher-dimensional lattices, hypercube, strongly regular graphs, Johnson graphs, "Search on Vertex-Transitive Graphs by Lackadaisical Quantum Walk," Rhodes and Wong (2020)

$$l = \frac{\text{degree}}{N}$$

Arc-transitive graphs, Yu and Høyer, 2020