Writing

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Abstract

Some unsolicited advice on writing papers.

1 Introduction

The purpose of a mathematical paper is to communicate interesting results to mathematicians, present and future. So, if you are going to write a paper, the first requirement is that you have some interesting results to present. If this is the case, I hope that you find the following advice useful.

2 Questions

There are three questions that the final draft of your paper should answer:

(a) How do your results and ideas relate to earlier work?

(b) Why should someone read your paper?

(c) What remains to be done?

To answer the first question, you will need a thorough knowledge of the related literature. You can provide a partial answer to the second by making clear what is new. (New results may well be interesting.)

3 The Plan

How do you start? Generally a paper consists of

(a) an abstract,

(b) an introduction,

(c) a number of sections in which preliminary results are developed and then the main results are proved,

(d) a list of references.

When you decide to write a paper, you will normally already have a collection of notes and ideas, which may reside in your head, on paper, or on a computer. You should think long and hard about how this material fits together and then prepare a brief but careful outline of the most important points. This might not be much more than a list of section headings. With this in place, you should then draw up a plan for each section. You will probably need to include outlines of the more complicated proofs, and you may find it useful to have a rough draft of an introduction. Once the structure of the paper is clear in your mind, start writing. It is best here to start from scratch, rather than attempt to recycle your notes.

With longer papers at least, you will probably find it easiest to write backwards: start with the sections giving your main results, then the preliminaries, then the introduction and finally the abstract. (It is often difficult to decide what preliminary material is needed until you have a good draft of the later sections.) As the writing progresses, flaws in your plan will become apparent. At this point you revise your plan, and then rewrite your latest attempt in accord with the changed plan. You may find that you have to revise your plan, and the paper itself, many times. Writing is a circular process.

4 Details

Abstract

The abstract should be as enticing as possible. It should use the absolute minimum of notation, and normally contain no definitions. It should be

entirely self-contained: if a reference to other work must be given, then the full citation should be provided. Substantial parts of it may be quotes from the introduction—this is not unnecessary repetition, because the abstract will be the only part of the paper most people read.

Introduction

The introduction is the next hurdle to be faced by those readers who were not turned off by your abstract. The role of the introduction is to state your main results, indicate the connections between your results and other work and to motivate the reader to study the rest of your paper. You should keep the introduction as simple as possible. It may be a good idea to state your results in a weaker form than you actually prove them—this may allow you to present them more clearly. A brief summary of earlier work related to yours should be given. This should not be taken as an opportunity to refer to every paper ever written by you or your friends.

You should explain carefully why your work is interesting. If you cannot do this, you should not be writing the paper. Unfortunately, the statements "I began work on this problem because it was given to me by my supervisor" or "I have previously solved the first 47 cases of this problem; here is the solution to the 48th" cannot be considered as providing motivation. One of them may well be the true reason why you started work, but they do not provide a reason why anyone else should look at what you have done.

It is usually necessary to include some definitions in the introduction. There is no harm in this at all. If your work raises some interesting open questions, some could be stated in the introduction. The introduction may conclude with a short description of how the rest of the paper is structured; this is more necessary in a longer paper.

The Meat

One frequently finds a section on notation and terminology following the introduction, and this could conceivably be necessary. Generally speaking though, terminology and definitions should be introduced as they are needed. Occasionally there will be some important term that is used in a number of places; this should be defined in some prominent place. Nonetheless, when you come to actually use it, you should assume the reader has forgotten the

definition. It is a kindness to repeat it, or at least offer precise directions to where it was given.

Proofs of theorems should always be immediately preceded by a complete statement of the theorem. If you do not do this, you may find that have applied a theorem when its hypotheses do not all hold. If you are lucky you may even notice this before you submit the paper.

It is essential that you use every means at your disposal to indicate what is important—basic formatting, the division into sections, the text itself. Something labelled as a theorem should be one of your principal conclusions. A lemma is a tool used to construct a theorem, and a corollary should be something that follows readily from a theorem. (Perhaps a corollary may be a corollary of the proof; but in this case it may be that the there is a good idea buried in the proof, and a corollary might not be the best way of disinterring it.)

Acknowledgments

Any acknowledgments can made at the end of the final section, or in a separate section just before the references. (They might also fit at the end of the introduction.) The key point concerning acknowledgments is that they should be clear, brief and to the point. Bear in mind that most of your readers are entirely uninterested in your private life; mathematics itself is enough of a struggle.

References

Finally there is the list of references. There are more mistakes made in this section than in any other. You can download the citations directly from MathSciNet; this will eliminate most sources of error. (This assumes you are using LaTeX.) I feel it is best to number the references, rather than index them by some combination of initial letters of the author's names. (It is easier to find a number in a list than 'GaGa'.) The citation should include the author(s) of the paper. For example, write "in Jones [18]", since this often allows the reader to infer what you are citing without working through the list of references. If you refer to an item in a book, you should state the number of the page, or theorem, or lemma you are referring to. If you cite a personal communication, then you should indicate in the text that it is a personal communication. There is no point in forcing the reader to go to

your list of references, just to find that they can go no further. (In fact I see little point adding items such as "J. Blake, personal communication" to the list of references.)

Other Stuff

There are many annoying little questions that arise about details of format (should I have space around this dash?,...). These can often be resolved by using a well-written paper as a model. The Chicago Manual of Style [1], or an equivalent, is a useful ultimate authority.

Some authors provide a final section where questions and conjectures raised by the work are discussed. This is an excellent idea. I do not feel there is any need for a formal summary of results at the end of the paper, as is the practice in some other areas of science. If the paper is well-written and has a good introduction, this should not be necessary.

5 By-Laws

1. Your overriding aim must be to make the reader's task as easy as possible. There may often be a choice between your convenience and the reader's; the reader's should come first every time.

2. **Plan:** There is a chess proverb "Even a bad plan is better than no plan". It applies to writing papers. In fact it applies to most things in life, but I digress.

3. You are **not** obliged to put down everything you know about your topic in the paper.

4. Use the layout of your paper to indicate the logical structure. The divisions into sections, paragraphs, etc. should be a reflection of the underlying logic (and not reveal its lack).

5. Keep the statements of your main results free from notation, and as clear as possible. Some readers will attempt to skim your main sections, using the statements of your main results as sign posts. Never include a definition in the statement of a theorem.

6. **Do not invent notation.**

7. Do not invent notation.

8. Well, you are only human, and you have this absolutely wonderful notational improvement, which must be released at once on an unsuspecting world. You should at least check that there is not a term already in use. If there is then you should use it instead. No one has ever gained even temporary fame in mathematics by introducing a new system of notation. You should also try to avoid the trap of believing that the notation used in your graduate school is the universal standard.

9. Cuteness is only tolerable in small children. This applies particularly to your choice of notation.

10. **It is better to be generalized than to generalize.** This is Erdős's law. Generality obtained at the expense of clarity is not worth the price.

11. **Defer, defer...** (W. S. Gilbert [2]) If you are writing a section and are not sure whether to include a particular lemma or definition at a given point, leave it till later. If the same question arises later, defer it again. Two times out of three you will find that the problem is eventually deferred to the list of references and thus it can be left out altogether. This is particularly important in dealing with definitions.

12. **Your text should be readable.** It should be possible to read it all in English, with notation translating consistently. So "$\in$" means either "is an element of" throughout, or "in", but not one or the other, according to the author's immediate need. (Halmos [4] stresses this point.)

13. Cut things out! It is amazing how often a difficult passage can be clarified by deleting words or phrases.

14. **Do not use code.** This could be called Halmos's law. As a first step, avoid abbreviations. Do not write a paper on "CPBNH-graphs". Such an abbreviation may save the author work, but it makes more work for the reader. Do not write "the # of such graphs on 7 vertices is = 3". As a general rule, you should keep the text as free of notation as possible. For more advice, read Halmos's article [4].

15. **Keep your thoughts in order.** "$A = B_{m,n}$, where n is ..., and m is ..., and if I had thought to define these cursed symbols before I used them

then I would not have become trapped in this long sentence that I do not see how to end."

16. **It is obvious that...** Expressions like this can be used, but only with great care. It is usually not obvious, and the experimental evidence indicates that the stated assertion is often false. If there is a two line reason, state it. (It is surprising how often this is possible.)

17. Do not be afraid to illustrate results and definitions by giving relevant examples. I have seen trivial papers padded out by simple examples worked out at great length, but generally there are too few, rather than too many.

18. "My results are important. The reader will not be put off by a few obscurities". There are two errors here. The syllogism is false, and there will also certainly be more than a few obscurities if this is your attitude.

19. Do not write a paper in the first person. If you need a pronoun, use "we". This means "the reader and I". When this is inappropriate ("we conjecture"?) you might use the first person. Use the first person when making acknowledgments.

20. Do not use contractions: we're, don't, wont and the like. You may feel that this creates an air of informality, rather than an air of inexperience. You are wrong. Spoken and written English are not the same, and ignoring this will prove distracting.

21. Do not start sentences with notation. Especially lower-case notation. You will also need to be careful with notation preceded by commas, because we use commas in mathematics to denote ordered pairs and so it may take some effort to decide whether a comma is mathematical or grammatical.

22. Punctuation is a mine-field. The purpose of punctuation is to help the reader parse your sentences. Thus commas do not necessarily correspond to the places where we would pause when reading the sentence aloud. To many commas can be as confusing as too few.

6 Examples—Bad Ones

The following is a direct quote. I have adjusted the line width to better approximate the appearance in the source.

> The fundamental notion is that of a von Neumann algebra. This is, typically, what may be called the 'symmetries of a group'. The precise way of saying this is that a (concrete) *von Neumann algebra* is nothing but a set of the form $M = \pi(G)'$ – where π is a unitary representation of a group G on a Hilbert space $\mathcal{H}$, and S' denotes, for S a subset of $\mathcal{L}(\mathcal{H})$ (the algebra of all bounded operators on $\mathcal{H}$), the commutant of S defined by $S' = \{x' \in \mathcal{L}(\mathcal{H}) : x'x = xx' \text{ for all } x \text{ in } S\}$. In other words M is the set of intertwiners of the representation π: thus, $x \in M \Leftrightarrow x\pi(g) = \pi(g)x$ for all g in G.

Aside from the fact that the expression "symmetries of a group" has very little meaning, there are no great problems with the first two sentences. Turning to the last sentence, if the coded expression

$$x \in M \Leftrightarrow x\pi(g) = \pi(g)x \text{ for all } g \text{ in } G$$

was truly necessary, it should be displayed (as I have just done). It is not truly necessary, we could recast the sentence thus:

> In other words, M is the set of elements in $\mathcal{L}(\mathcal{H})$ that commute with the operators $\pi(g)$, for all g in G.

But we have left the worst till last. Even with mathematical text, it should be possible to read it aloud from beginning to end, and have an interested listener successfully comprehend what you have said. The third sentence is a true horror, and fails this and any other rational test.

First, a comparatively small point. The 'nothing but' in the third does nothing but annoy the average reader. (Both of the colleagues I showed this paragraph to remarked on this, without any prompting from me.)

Although it is probably not their intention, authors often appear to use parentheses to indicate afterthoughts. Afterthoughts have no place in the final version of a well-written paper. The phrase "the algebra of bounded operators on $\mathcal{H}$" is a good example. Terms should be defined before they are

used. You can avoid parentheses by preceding the expression with 'where', but this is not better.

The problem with the "(concrete)" is different—the difficulty is that no mortal could decide what 'concrete' is intended to mean here, without reading the sentence several times. (A cruel and unusual punishment.) The authors could have done worse. Sometimes one meets expressions of the type "a hot (cold) bun is a bun having a high (low) temperature". Here the objective appears to be to save characters of text, rather than to make things easy for the reader.

We turn to the final phrase, starting "the commutant of S defined by...". The first problem here is that the "commutant of S" refers back to something before the 'parenthetic interjection', while "defined by" is referring to the encoded definition that follows. There is no simple fix for this. The definition would probably be better decoded, as I demonstrated with the last sentence. Note that the line break occurs in the same place in the original as in our quote. The authors are lucky, it could have occurred at a worse place. They could, of course, have chosen to display the expression. But decoding is better.

We offer the following translation of the entire quote:

> The fundamental notion is that of a von Neumann algebra. The basic example of this is $\mathcal{L}(\mathcal{H})$, the algebra of all bounded operators on $\mathcal{H}$. If π is a unitary representation of a group G then a *concrete von Neumann algebra* is the set of elements of $\mathcal{L}(\mathcal{H})$ that commute with each of the operators $\pi(g)$, for g in G. Thus, if we define the commutant S' of a subset S of $\mathcal{L}(\mathcal{H})$ to be the set of operators that commute with each element of S, then a concrete von Neumann algebra is the commutant of $\pi(G)$.

A second example:

> For $|A| \leq r$ define
> $$B(A) = \sum_{A \subset F} \alpha(F). \tag{1}$$
> Set $B = [k + r] - G$, $|B| = r$. Define
> $$\delta(B) = \sum_{B \cap F = \emptyset} \alpha(F). \tag{2}$$

By inclusion-exclusion we have

$$\delta(B) = \sum_{A \subset B} (-1)^{|A|} B(A). \tag{3}$$

If $q(\mathbf{x}) \in V_r$ then $B(A) = 0$ and thus $\delta(B) = 0$ follows.

The main problem here is that the 'B' in (1) is not the same as the 'B' in (2); hence the two 'B's in (3) and the two in the line following are not equal. However there are also a number of minor problems.

We remarked above that it should always be possible to read your text aloud, and that the result should be comprehensible. Also, as Halmos [4] forcefully argues, your notation should always translate to the same string of words. For example, if you think '$\leq$' means "is less than or equal to" then the first part of the first line above must read "For the size of A is less than or equal to r define...". (I agree entirely that following this rule makes life harder for a writer.)

The sentence "Set $B = [k + r] - G$, $|B| = r$" suffers the same problem—lack of a consistent English translation. Also the string $|B| = r$ reads as if we are defining $|B|$, not r. Finally the substring "G, $|B|$" has notation separated by punctuation. This is not a particularly bad example of this crime, but it is still makes the sentence harder to parse. One reasonable alternative formulation is: "Suppose $B = [k + r] - G$ and $r = |B|$".

There are no significant problems with the final sentence.

7 TEXnical Stuff

I recommend you find an editor that will make it easy for you to write LaTeX, python code (for example) and html. It should take care of balancing parentheses and the like.

You will have to learn to use LaTeX well. The most useful reference is probably Grätzer [3] or, for example, Helmut and Daly [5]. I have found that google generally leads me quickly to good advice.

The comments that follow will only make sense if you have some experience with LaTeX.

You should try to keep the input file as readable as possible, and similar in structure to the output. For example, in the input file I always display

anything which will be displayed in the output. In comparison with handwritten work, you will need to display things much more often. As a general rule, try to keep the text as free from symbols as you can—in other words, most of the symbolic soup should be displayed.

Next, LaTeX is much better at page layout than you are. Try to work with the defaults as much as possible, and only change the spacing between words or lines when something is clearly broken. Lamport asserts the first, second and third most common mistakes in using LaTeX are the same—worrying too much about format and not enough about content.

Do not set the page size, line lengths and the like yourself. These are determined by the document class.

You will eventually build up a file of macros, with which you will continually tinker. And then, one day, you will try to run off a copy of an older paper and discover that it uses macros that are now lost! When your paper is in final form, insert all the macros it uses at the front of the file.

Preparing solutions to course homework is one of the more difficult things to do well in LaTeX. An article of 10 pages would be easier.

Preparing figures for a paper is still a pain. You should aim to keep figures simple; complicated diagrams are not often worth the time needed to prepare them. The standard tool currently is tikz; if you are starting out it might be worth the effort required to learn how to use it.

8 Submission

Having written the paper, it is natural to want to have it published. As writing a good paper requires an effective knowledge of the literature, you should already know a number of journals which have published papers on topics similar to yours. One of these is the obvious place. Go to the journal web page and read the "instructions to the authors". Probably the most important part of this is the advice on where to actually send the manuscripts. Having submitted your manuscript you should receive an acknowledgement of receipt. If you do not receive this within a couple of weeks, then write and ask if it has been received.

After this you are at the mercy of the referee(s). It is reasonable, and recommended, to ask the editors about your paper after six months have gone by and, in most cases, you should have a final decision within a year. If your paper is accepted as is, there is no need to say more.

More often the acceptance is conditional on you revising your paper in the light of the referees' reports. Occasionally they will offer advice which you think is wrong. It may be necessary to ignore it but then, when you resubmit your paper, your covering letter should state clearly where you have ignored the referee's advice, and why. You must not add new material at this point. If you make any changes other than those recommended in the reports, you should point this out in your covering letter.

If your paper is rejected, you should carefully read the comments provided by the referees, and use them as a guide in revising your paper. You can then submit your paper again, to a different journal. There is a good chance it will end up with the same referee; if the referee was unhappy the first time, they will be even less happy to see it a second time, unaltered. I have seen authors waste considerable amounts of time in trying to get a poor quality paper published.

Once you have submitted a paper to a journal, you cannot submit it to another journal unless the first journal rejects it, or you have written to the first journal asking to withdraw it. (Ignoring this could be a career-wrecking mistake!)

Note that the decision to accept or reject a paper is made by the editors, in the light of the reports they have received from referees. The editor may reject a paper even when the reports are positive. If you are lucky, the reasons for rejecting a paper will be clear from the referees' reports. There is very little to be gained by writing to the editors to argue about comments made by the referees. (The problem is that the editor will usually have more confidence in the referee's judgement than in yours. And good referees are much more important to editors than mere authors.)

9 Mathjax

It can be useful to produce short mathematical documents to be read using a browser. For this purpose pdf files are less than ideal, but there is a workable alternative: mathjax in combination with html. My view is that for short documents, this combination is already the best way to proceed.

One problem is that now you need an editor that is both html-aware and latex-aware, and I do not know of such a beast. My current approach is to work in the html mode of my editor (the latex you use is quite simple, with no formatting commands).

The are programs that will work with mathjax and markdown simultaneously. In this context though, I think markdown is a poor substitute for a proper editor. One difficulty is that each of these programs has its own conventions for delimiting tex, and so files cannot be transferred from one program to another (and your coauthor will doubtless choose an incompatible program).

Other Sources

You will need a good dictionary. Halmos's article [4] should be compulsory reading. For actual useful advice on writing well, there is Williams [9]. For an authorative grammar guide, I recommend Huddleston and Pullum's "The Cambridge Grammar of the English Language" [7]. The "Chicago Manual of Style" [1] is an obvious source of help. (With the last two tomes at hand, you should be able to hold your own in any argument with an editor or colleague.)

It is traditional in guides such as this one to point to Strunk and White. This not a useful guide, and should be ignored. (See `http://www.lel.ed.ac.uk/~gpullum/50years.pdf` for informed, trenchant, criticism.) Similarly Truss [8], although entertaining, is often wrong. Most of the good advice in Higham [6] can be found in Halmos; I disagree with much of Higham's advice on TEXnical matters. (One point of this paragraph is that there is a lot of authoratively presented bad advice available, if you're in a hurry.)

References

[1] *The Chicago Manual of Style*, U. Chicago Press, Chicago, 1993.

[2] W. S. Gilbert. *The Mikado.*

[3] G. Grätzer. *Math into LaTeX,* Birkhäuser, Boston, 2000.

[4] P. Halmos. *How to write mathematics,* L'Enseignement Math, **16** (1979), 123–152. `https://www.mathematik.uni-marburg.de/~agricola/material/halmos.pdf`

[5] K. Helmut, P. W. Daly. *A Guide to LaTeX2ε,* Addison-Wesley, Harlow, 1993.

[6] N. J. Higham. *Handbook of Writing for the Mathematical Sciences,* 2nd edition, SIAM, Philadelphia 1998.

[7] Rodney Huddleston, Geoffrey K. Pullum. *The Cambridge Grammar of the English Language*, Cambridge U. P., Cambridge, 2008.

[8] L. Truss. *Eats, Shoots & Leaves*, Gotham, 2003.

[9] J. M. Williams. *Style: Towards Clarity and Grace*, U. Chicago Press, Chicago, 1990.