MATH 650 : Hints and Suggestions for Written Assignment 1

In solving linear, separable, or exact DEs, it is expected that you will use the step-by-step procedures discussed in lectures, and present your solutions in that format. (See also the Sample Solutions)

1. In part a. you could solve the DE in general, then apply the ICs. For b., recall the behaviour of t^a for real $a > 0$ and $a < 0$.

2.c. Remember that $t_0 = 0$ must be inside the interval of validity; solutions cannot 'cross over' asymptotes! Use Maple to plot sketches for part **d.**.

3. Some DEs may be of more than one type; some may be solved by more than one method.

4. Note that the DE revised to standard form is not defined for all (x, y). Also, remember that the initial value x_0 must be contained in the domain of validity of the solution of the IVP.

5.b. As in problem 4, you will find that one value of the constant of integration gives more than one explicit solution. Be careful to pick the one(s) that corresponds to the IC(s).

6.b. Note that it is convenient to re-set the time to 0 when the soup is moved to the counter.)

7. The solution in part **c.** will look a bit messy...don't expand it!

8.a.,b. Remember that the half-life T_h is the time when only half the initial quantity is left.

9. Label the quantities r_{in}, c_{in}, r_{out} carefully, guided by the basic mixing model.

10. Consider what happens in a short interval Δt: there is a volume of fluid gained from the inflow, and a volume of fluid lost from the hole. The net gain is the difference, and must equal $A\Delta h$. For part d., recall the inverse derivative relationship which implies that $\frac{dt}{dh}$ is the reciprocal of $\frac{dh}{dt}$. Let $t*$ be the time it takes for the tank to empty, and note that as t goes fom 0 to $t*$, h goes from 3 m to 0.

11.d. Once you find the expression for $C(t)$, differentiate. You should get a rational function with lots of obviously positive terms. Work with the remaining term in the numerator to show that $r \geq 0.0012$ per day guarantees $C'(t) \geq 0$ for all $t \geq 0$.

12. Do part a. carefully, since the rest of the problem depends on the solution. In part b., you'll need to do a bit of algeba. For c., see the Maple file with these Hints.

13.a.,b.,e. Note the orientation here: positive displacement DOWNWARD. Do the algebra and the chain rule carefully in part b.; you should get $\dfrac{dv}{dt} = g\dfrac{du}{d\tau}$ via the chain rule. You will need to find $v(10)$ to do part e.

14. This problem essentially parallels the Sunken Treasure example, except that the drag force is proportional to v^2. Be careful with the signs on the various forces in setting up your model. The numbers are a bit messy, so use a calculator here.

15. No solutions of DEs are needed in this problem; all your aguments should be based on your knowledge of logistic growth and its graph, and on examining the sign of the derivative (i.e., qualitative analysis).

16. For a ii. and iii., and b iii., follow the direction field analysis steps as done for logistic growth in Module 3. Then draw careful sketches reflecting your observations. For b i., use your prior knowledge of logistic growth. Again, NO ANALYTICAL SOLUTIONS are needed in this problem. NOTE: The RHS of the DE factors nicely in b iii. DON'T FORGET the phase lines in a iii. and b iii.; show them vertically alongside your solution sketches.

17. It would be helpful to review what you know about Euler's Method before starting this problem. In part f ii., you should get a theoretical upper bound on $|E_5| \leq 43.2$ on $0 \leq t \leq 1$.

18. In part (a.), recall what it means if the derivative of an expression is zero. In part (b.), you will get two cases from taking the square root; keep the signs straight as you complete the solutions.