# MATH 650 : Mathematical Modeling

## Spring, 2019

## Electronic Assignment #8

### Due by 11:59 p.m. EST on Tuesday, July 16, 2019

**Instructions:**

- Ensure you have reviewed Module 7, Sections 3, 4, and 5, and any other activities therein. You will require this knowledge to answer the questions on this assignment.
- Read and think about the following assignment problems.
- Print and complete the following assignment. Record your answers on the printed copy so you have a record of your solutions.
- Once you are satisfied with your answers, submit your solutions online as follows:
  - Go to UW's course management website at `learn.uwaterloo.ca`
  - Enter your **QUEST Username** and **Password** in the space provided and click **Login**.
  - Once inside the LEARN course environment, click on the link for **MATH 650 : Mathematical Modeling**.
  - Click on the **Submit** → **Quizzes** tab at the top of the page.
  - Click **Electronic Assignment 8**, and follow the instructions provided. An answer key for this assignment will appear where you can fill-in your solutions. Please email your instructor immediately if you encounter any problems.
  - Click on the **SUBMIT QUIZ** button when you are done. You have only 1 attempt to submit your solutions. Any assignment submitted after midnight (Waterloo, Ontario time) will be considered **late** and will not be counted toward your final grade (no exceptions).

**Note that this assignment has 17 questions, 12 True/False and 5 Multiple Choice.**

**The following questions are based on Module 7, Sections 3, 4, and 5**

**Part 1: True or False** (1 mark each)

Indicate whether the following statements are true (a) or false (b).

**1.** For the DE $y'' + y' + y = 2\cos t + t^2 + e^{-t}$, the assumed form of the particular solution should be $y_p = A\cos t + Bt^2 + Ct + De^{-t}$, where $A, B, C, D$ are coefficients to be determined.

**a.** True

**b.** False

**2.** A particular solution of the DE $y''+y = \ln t$ can be found by the method of undetermined coefficients. [HINT: Don't try to solve this DE; just look at the right hand side.]

**a.** True

**b.** False

**3.** The solution of the IVP $y'' + 5y' + 6y = 10e^{2t},\ y(0) = 1,\ y'(0) = 1$ is $y(t) = \frac{3}{2}e^{-2t} + e^{-3t} + \frac{1}{2}e^{2t}$. [SUGGESTION: Either solve the IVP, or just see whether the given solution satisfies the DE and ICs, whichever you think is more efficient.]

**a.** True

**b.** False

**4.** If $y_{p_1} = -\frac{1}{2}\cos t$ is a particular solution of $y'' - y = \cos t$, and $y_{p_2} = -2t^2 - 4$ is a particular solution of $y''-y = 2t^2$, then the general solution of $y''-y = \cos t+2t^2$ is $y(t) = c_1e^{-t}+c_2e^t-\frac{1}{2}\cos t-2t^2-4$.
[HINT: Think about nonhomogeneous superposition...]

**a.** True

**b.** False

**5.** The amplitude of the steady-state solution of the DE $y'' + \frac{1}{8}y' + y = 3\cos t$ is $A_{ss} = 24\,\mathrm{m}$.

**a.** True

**b.** False

**6.** The DE $y'' + y' = 3e^{-t}$ has a particular solution of the form $y_p = Ate^{-t}$ for some constant $A$.

**a.** True

**b.** False

**7.** Knowing that a particular solution of the DE $y'' + ay' + by = 2\cos t + \sin t$ is $y_p = 2\sin t$ is NOT sufficient to determine the values of the constants $a$ and $b$.
[HINT: Try to find $a$ and $b$ by substituting the given solution in the DE.]

**a.** True

**b.** False

**8.** To apply the method of variation of parameters to find a solution $y = v_1y_1 + v_2y_2$ of the DE $y'' + y = \ln t$, we need to solve the equations $v_1' \sin t + v_2' \cos t = 0$ and $v_1' \cos t - v_2' \sin t = \ln t$ for $v_1'$ and $v_2'$, and then integrate to find $v_1$ and $v_2$.
[HINT: What are the solutions $y_1$ and $y_2$ of the homogeneous DE? NO NEED TO SOLVE the DE.]

**a.** True

**b.** False

**9.** Once we have found the fundamental matrix $\mathbf{\Phi}(t)$ for the homogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x}$, the solution for any initial condition $\mathbf{x}(0) = \mathbf{a}$ is $\mathbf{x}_h(t) = \mathbf{\Phi}(t)\mathbf{a}$.

**a.** True

**b.** False

**10.** The long-term (steady-state) behaviour of a mass-spring-damper system with periodic forcing is always periodic because the transient (homogeneous) solution is also periodic.

**a.** True

**b.** False

**11.** In Example 7.4.2, the Bouncy Ride, the model predicts that the amplitude $\mathcal{A}_{ss}$ of the 'bounces' (the car's response) is directly proportional to the amplitude (height) of the bumps in the road, and decreases as their frequency increases.

**a.** True

**b.** False

**12.** For a periodically forced system $my'' + \gamma y' + ky = F\cos(\omega t)$, the steady-state amplitude $\mathcal{A}_{ss}$ can achieve a maxiumum (amplitude resonance) only if the damping coefficient $\gamma$ is sufficiently small, specifically, if $\gamma < \sqrt{2km}$ (in terms of the original physical parameters).
[HINT: See Exercise 7.4.2 in the course lectures.]

**a.** True

**b.** False

**Part 2: Multiple Choice** (1 mark each)

Choose the **best** answer for each question.

**13.** The system $y'' + 4y = 3\cos(\omega t)$
[SUGGESTION: First read pages 269-271 of your text.]

**a.** will have bounded solutions for any $\omega \geq 0$;

**b.** will exhibit the phenomenon of *beats* for $\omega = 2.1\,\mathrm{s}^{-1}$;

**c.** will have unbounded solutions for $\omega = 2$.

**d.** Two of the above statements are true.

**e.** None of the above

**14.** Find the general solution of the DE $y'' + 2y' - 3y = 2\cos t$. Then decide which statements are true.
[HINT: Find equations relating $y_0$ and $v_0$ to the constants of integration.]

**a.** Most solutions are unbounded as $t \to \infty$.

**b.** All solutions with initial conditions $y(0) = y_0$, $y'(0) = v_0$ satisfying $3y_0 + v_0 = -1$ are bounded on $t \geq 0$.

**c.** The solution with initial conditions $y_0 = -\frac{2}{5}$ and $v_0 = \frac{1}{5}$ is periodic.

**d.** All of the above

**e.** None of the above

**15.** For the mass-spring-damper system with periodic forcing, $my'' + \gamma y' + ky = F\cos(\omega t)$, the steady-state response $A\cos(\omega t - \phi)$

**a.** has phase angle $\phi$ defined on $0 \le \phi \le 2\pi$;

**b.** is never out of phase with the input;

**c.** lags the input by $\frac{\pi}{2}$ when the input frequency $\omega$ matches the natural frequency $\omega_0$.

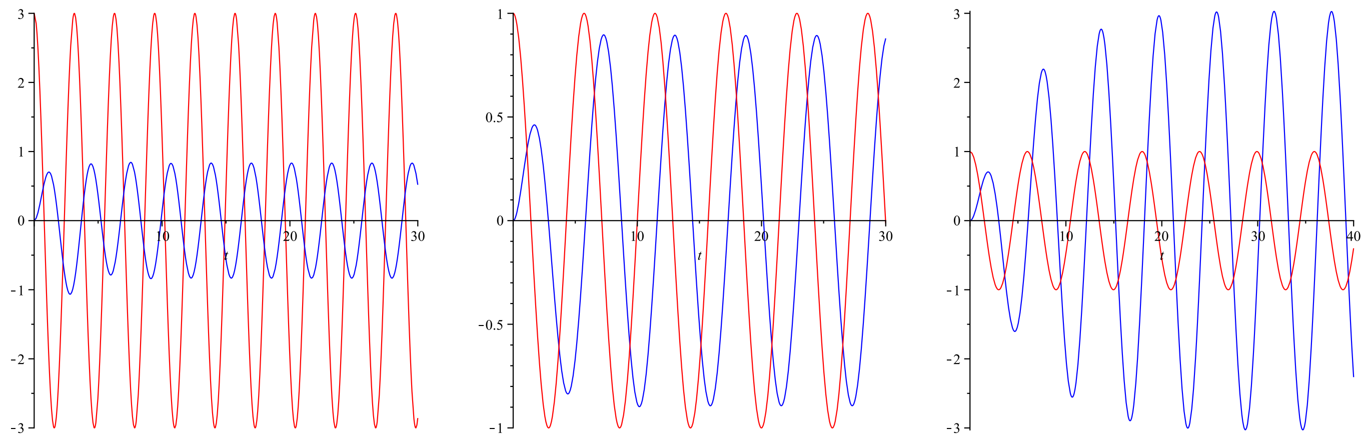
**d.** All of the above

**e.** None of the above

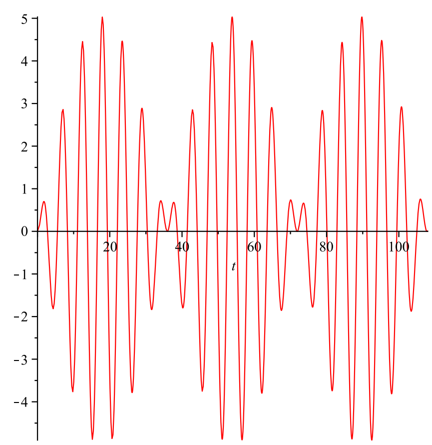
**16.** If the blue curve in each plot below is a solution of a periodically forced system $y'' + 2\delta y' + \omega_0^2 y = A\cos(\omega t)$, and the red curve is $A\cos(\omega t)$, which of the systems 1, 2, or 3 displays *amplitude resonance*?

**a.** System 1 **b.** System 2 **c.** System 3 **d.** All three systems **e.** None of the systems

**System 1** **System 2** **System 3**




**17.** The diagram below displays a solution of a certain undamped, periodically forced system.




**a.** This graph represents the phenomenon known as *beats*.

**b.** This graph represents the phenomenon known as *resonance*.

**c.** This graph represents the response of the system when the frequency of the periodic forcing is close to, but not equal to, the natural frequency of the system.

**d.** Only statement a. is true.

**e.** Only statements a. and c. are true.