MATH 650 : Mathematical Modeling

Spring, 2019

Electronic Assignment #1

Due by 11:59 p.m. EST on Tuesday, May 14, 2019

Instructions:

- Ensure you have reviewed Module 1, Sections 1 and 2, and any Text Readings and other activities therein. You will require this information to answer the questions on this assignment.
- Read and think about the following assignment problems.
- Print and complete the following assignment. Record your answers on the printed copy so you have a record of your solutions.
- Once you are satisfied with your answers, submit your solutions online as follows:
  - Go to UW's course management website at `learn.uwaterloo.ca`
  - Enter your **QUEST Username** and **Password** in the space provided and click **Login**.
  - Once inside the LEARN course environment, click on the link for **MATH 650 : Mathematical Modeling**.
  - Click on the **Submit** → **Quizzes** tab at the top of the page.
  - Click on **Electronic Assignment 1**, and follow the instructions provided. An answer key for this assignment will appear where you can fill-in your solutions. Please email your instructor immediately if you encounter any problems.
  - Click on the **SUBMIT QUIZ** button when you are done. You have only 1 attempt to submit your solutions. Any assignment submitted after midnight (Waterloo, Ontario time) will be considered **late** and will not be counted toward your final grade (no exceptions).

The following questions are based on material in Module 1, Sections 1 and 2.

Part 1: True or False (1 mark each)

Indicate whether the following statements are true (a) or false (b).

1. Interpreting what a model predicts is the final step in the modelling process, and may sometimes indicate that we need to modify the model.

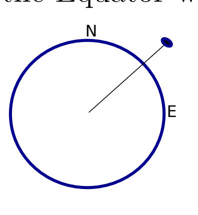
a. True

b. False

2. An astronaut aboard the International Space Station at altitude 350 km will be able to see both the North (geometric) Pole and the Equator when passing above 45° N. latitude.

a. True

b. False



3. Because we neglected haze, our model will predict a horizon distance x which is less than the actual horizon distance.

a. True

b. False

4. If gravity is the only force acting on two bodies of equal mass m, Newton's Laws of Gravity and Motion predict that *each* body will move with velocity v according to the DE $\frac{dv}{dt} = -\frac{Gm}{r^2}$, where r is the distance between them, and the origin of the frame of reference is at the centre of mass of the other body.

a. True

b. False

5. The gravitational constant g is the gravitational force on an object in motion near the Earth's surface.

a. True

b. False

6. Neglecting resistive forces, the equation of motion for an object moving with velocity v in the Earth's gravitational field is equivalent to $\frac{dv}{dt} = -g\frac{1}{(1+\frac{s}{R})^2}$, where $-g$ is the vertical acceleration near the Earth's surface, R is the Earth's radius, and s is the vertical displacement upward.

a. True

b. False

7. With Newtonian damping, and displacement measured positive upward, the equation of motion for a body of mass m, in motion near the Earth's surface with vertical velocity $v < 0$, is $\frac{dv}{dt} = -g + \frac{\beta}{m}v^2$, where β is a positive constant.

a. True

b. False

8. Resistive forces (i.e., drag forces) always act in the same direction as velocity.

a. True

b. False

9. In the basic population model $\frac{dp}{dt} = rp - K$, one of the contributing factors to the constant K is the per capita birth rate.

a. True

b. False

10. If the distance from the Sun to the Moon is about the same as that from the Sun to the Earth, then the approximate ratio of the gravitational force of the Sun on the Moon to that of the Earth on the Moon, according to the given data, is about 2.20.

GIVEN DATA: mass of the Earth $= 5.972 \times 10^{24}$ kg, mass of the Sun $= 1.989 \times 10^{30}$ kg, average distance from the Earth to the Moon $= 3.844 \times 10^{8}$ m, average distance from the Earth to the Sun $= 1.496 \times 10^{11}$ m.

a. True

b. False

11. For a teapot with a tea cozy (a padded cover), the transmission coefficient k is smaller than for an uncovered teapot.

a. True

b. False

12. Suppose that, in Newton's Law of Temperature Change $\frac{du}{dt} = -k(u - T(t))$, the ambient temperature has the form $T(t) = B + A\sin(\omega t)$. Then the solution for large t is also sinusoidal, with the same frequency ω.

a. True

b. False

13. The solution of the basic IVP $\frac{dy}{dt} = a(y - y_e)$, $y(0) = y_0$ is never a constant function.

a. True

b. False

14. If a DE $\frac{dy}{dt} = f(t, y)$ has an equilibrium solution y_e, then nearby solutions always converge to this equilibrium solution as time increases.

a. True

b. False

Part 2: Multiple Choice (1 mark each)

Choose the **best** answer for each question.

15. For a DE $\frac{dy}{dt} = f(t, y)$, a solution satisfying $\frac{dy}{dt} = 0$, i.e., $f(t, y_e) = 0$, is called an *equilibrium solution*.
Consider the population IVP $\frac{dp}{dt} = rp - K$, $p(0) = p_0$.

a. The equilibrium solution is $p_e = \frac{K}{r}$.

b. Higher immigration i gives a higher equilibrium solution.

c. A higher birth rate b gives a higher equilibrium solution.

d. An initial state $p_0 < \frac{K}{r}$ causes extinction in a finite time.

e. Exactly two of the above statements are true.

16. Suppose that a bird is flying 10 meters above the ground. Predict the approximate horizon distance.

a. 127 km

b. 100 km

c. 18 km

d. 11290 m

e. None of the above

17. Which of the following is a solution of the differential equation $\frac{dy}{dt} + 4y = 4$?

a. $y = 4t$

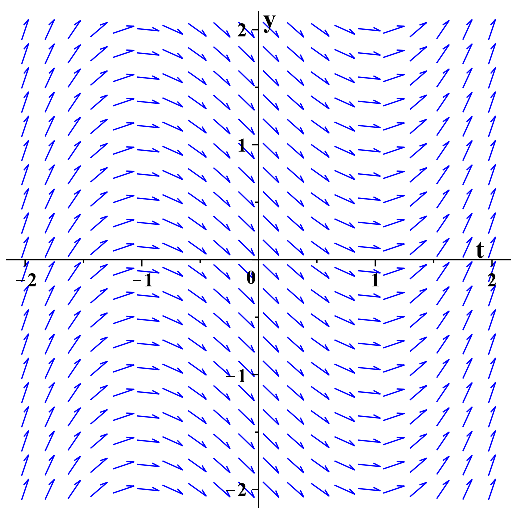
b. $y = 2t^2$

c. $y = e^{4t} - 1$

d. $y = c\,e^{-4t} + 1$ for any constant c

e. None of the above

18. A direction field is given on the right below.



Which of the following could be the corresponding DE?

a. $y' = y^2 - 1$

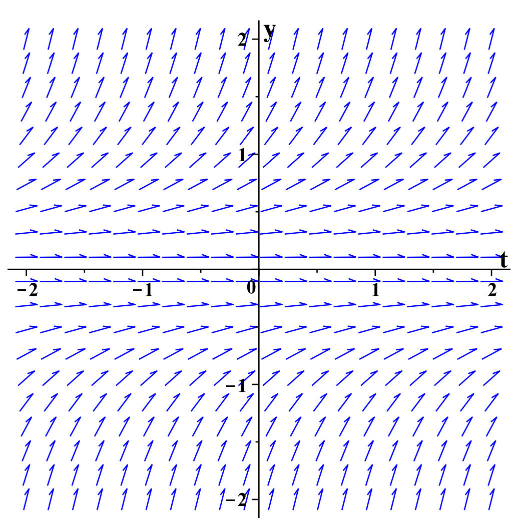
b. $y' = 1$

c. $y' = t^2 - 1$

d. $y' = t^2 + y^2$

e. None of the above

19. A direction field is given on the right below.



Which of the following best represents the corresponding DE?
[Hint: Are there equilibrium solutions?]

a. $y' = t^2$

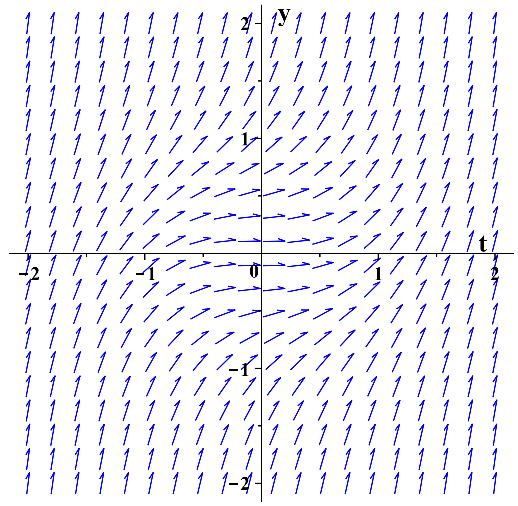
b. $y' = 1$

c. $y' = y^2 - 1$

d. $y' = t^2 + y^2$

e. None of the above

20. A direction field is given on the right below.



Which of the following best represents the corresponding DE?

a. $y' = t^2$

b. $y' = 1$

c. $y' = y^2$

d. $y' = t + y^2$

e. None of the above

21. For a fluffy dandelion seed falling through the air, an appropriate assumption is that the magnitude of the resistive force is

a. proportional to its speed $|v|$.

b. proportional to v^2.

c. negligible.

d. proportional to its velocity v.

e. None of the above

22. Please refer to Web Activity 1.2.1 for this question. What is the maximum range x_f for $y = 50$?

a. 150 m

b. 136 m

c. 124 m

d. 96 m

e. None of the above

23. For the DE $\dfrac{dy}{dt} = 1 + 3y$,

a. a family of solutions is $y = Ce^{3t} + 1$.

b. one solution is $y = e^{3t} + \frac{1}{3}$.

c. all solutions approach the equilibrium solution as $t \to \infty$.

d. the equilibrium solution is not the limit as $t \to \infty$ of any other solution.

e. None of the above

24. For the DE $\dfrac{dy}{dt} = -2y + 10$,

a. any function $y(t) = 5 + Ce^{-2t}$ for an arbitrary constant C is a solution.

b. there is one equilibrium solution, $y_e = 5$.

c. for $y(0) = 1$, the solution is $y = 5 - 4e^{-2t}$.

d. the direction field has positive slopes for all $y < 5$.

e. All of the above

25. In the model for projectile motion, with initial height y_0, initial angle θ_0, and initial speed v_0,

a. for $y_0 = 0$, the range (landing point) is $x_f = \dfrac{v_0^2 \sin 2\theta_0}{g}$, and hence x_f is a maximum if $\theta_0 = \frac{\pi}{4}$.

b. the landing time is $t = \dfrac{v_0 \sin\theta_0}{g} + \dfrac{\sqrt{v_0^2 \sin^2\theta_0 + 2gy_0}}{g}$.

c. for fixed y_0, the landing time is greatest for vertical motion (i.e., when the object is tossed straight up).

d. All of the above

e. None of the above