Instantaneous Velocity

Created by

Barbara Forrest and Brian Forrest
Problem:
A stone is thrown straight upward in the air and eventually falls back to the ground. How can we define the *instantaneous velocity* of the stone at any given time?
**Instantaneous Velocity**

\[ m = v_{ave} = \frac{s(t_1) - s(t_0)}{t_1 - t_0} \]

**Recall:** The *average velocity* of the stone relative to the ground over the period from time \( t = t_0 \) to \( t = t_1 \) is given by the formula

\[ V_{ave} = \frac{\text{displacement (change in position)}}{\text{elapsed time}} = \frac{s(t_1) - s(t_0)}{t_1 - t_0} = \frac{\Delta s}{\Delta t} \]

where \( \Delta s = s(t_1) - s(t_0) \) and \( \Delta t = t_1 - t_0 \).
**Instantaneous Velocity**

\[ m = v_{\text{ave}} = \frac{s(t_1) - s(t_0)}{t_1 - t_0} \]

**Geometric Interpretation:** \( V_{\text{ave}} \) is the slope \( m \) of the “secant line” to the graph of \( s(t) \) through the points \((t_0, s(t_0))\) and \((t_1, s(t_1))\).
Instantaneous Velocity

$m = \bar{v} = \frac{s(t_1) - s(t_0)}{t_1 - t_0}$

$s = s(t)$

Question: How do we define instantaneous velocity at a point $t_0$?
Instantaneous Velocity

Key Assumption: The velocity of the stone should not vary too much over very small intervals of time. Therefore, if $h$ is small

\[
v(t_0) \cong v_{\text{ave}} = \frac{s(t_0 + h) - s(t_0)}{h}
\]
Definition: [Instantaneous Velocity]

The *instantaneous velocity* of an object at time $t_0$ is given by

$$v(t_0) = \lim_{h \to 0} \frac{s(t_0 + h) - s(t_0)}{h}$$

provided this limit exists.