Mat 223 — Linear Algebra I — Fall 2020

Textbook: *Linear Algebra: MAT223 Workbook*   Tutorials: Starting Monday, Sep. 27

Webpage: `http://q.utoronto.ca`

Linear algebra is the study of vectors, "flat spaces" like lines and planes, and linear transformations like rotations and scalings. Vectors originated in the study of physics and the 3D world, but through the mathematical practice of *abstraction*, we now use vectors in non-spatial realms, like music, computer graphics, and the study of physical forces.

Transformations are functions that move vectors around, and in this class we will focus on *linear transformations*. Why? Because we have a complete theory of linear functions. And, although mankind has tried to understand the non-linear phenomena of the universe, we haven't gotten very far. Despite 200 years of effort, the non-linear equations governing fluid flow haven't been solved! Because of this, our approach to answering general questions about the universe is often to convert the problems into linear ones—ones that we can actually understand, and ones which we will study in this course.

This course will cover the basics of linear algebra. We will ground our study in $\mathbb{R}^n$ (n-dimensional Euclidean space), using spatial intuition to guide us. However, we will also pay close attention to the mathematical definitions we encounter along the way. These carefully constructed definitions—the result of hundreds of years of human endeavor—will allow us to solve problems where our intuition fails (for example, how can you find the angle between two 17-dimensional vectors?). In the next course, Mat 224, the idea of vectors themselves will be decoupled from Euclidean space and linear algebra will become even more broadly applicable. But, for now, we'll start with the basics.

Learning Outcomes

After taking this course, you will be able to:

- Work independently to understand concepts and procedures that have not been previously explained to you.
- Clearly and correctly express the mathematical ideas of linear algebra to others, and understand and apply logical arguments and definitions that have been written by others.
- Translate between algebraic and geometric viewpoints to solve problems.
- Use matrices, matrix arithmetic, matrix inverses, systems of linear equations, row reduction, determinants, and eigenvalues and eigenvectors to solve problems. As well, write vectors in different bases and pick an appropriate basis when working on problems.

To Succeed

Learning is hard! It is exercise for the mind, and like exercise, when you're doing it, it feels pretty uncomfortable. Here are some tips to help you succeed academically (getting the grade you want) and intellectually (learning the most you can).

- Form a regularly-meeting study group of 3–4 people. Math should not be done in isolation! You need others to bounce ideas off of and to motivate you when you're feeling down.
- Read the textbook *before* class. A good rule of thumb is *every hour spent studying before class is worth two hours of studying after class.*
- Force yourself to explain. When reviewing, it's easy to glance at a solution and think, "Oh yeah, I knew that." Don't do it! Force yourself to explain each problem/concept without referencing a solution. Be patient with yourself—it might take 5 or 10 minutes before it finally comes to mind, but studying this way will be significantly more effective.

Prereq's

To be prepared for this course, you need to have a solid understanding of highschool mathematics, especially functions and function composition. You should also understand what a *solution* to an equation is (especially equations involving multiple variables).

None of the *procedures* in this course are difficult, but the *concepts* will be more abstract than highschool mathematics and will require mathematical habits of mind: dedication, ability to decide whether you're right or wrong (without referring to a textbook/answer key), precision (you can carry out dozens of mathematical steps without making a mistake), etc..

Homework

Linear Algebra is a mix of abstract concepts and concrete algorithms, and knowledge of both the algorithms and the concepts is essential to becoming a practitioner. However, the algorithms are the easiest part of this course. As such, lecture time will focus on the difficult concepts and you will be expected to learn and practice the algorithms for homework.

Online Homework: Each week (except for reading week), there will be a video playlist explaining a new linear-algebra procedure. There will also be an online homework where you will get a chance to practice this procedure as well as concepts learned in class. These homeworks should be completed by *Sunday at 11:59pm* each week. The details of each assignment will be posted on the course webpage.

Written Homework (PAR): A big focus of this course is mathematical communication. We will be working with you to improve your writing and communication. Effective writing and communication is hard, but it's invaluable in all technical careers (no matter what your career, you will have to defend products or ideas, make proposals, and explain what you do).

Part of your communication training consists of reading, reflection, and presentations. We will use the *Peer-Assisted Reflection* (abbreviated *PAR*) model in this course. For the three PAR assignments this semester, you will work with a partner to craft a draft response. Then, you will share your response and get feedback on your writing. With that feedback, you will revise your draft and submit your final copy as well as your draft.

Practice Problems: To do well in this course, you must practice! Each week, you should complete *all* practice problems listed at the end of the week's assigned module(s). These problems will be closer to midterm questions and will often ask you to combine many ideas used in class. Even though these problems will not be marked, it is essential that you practice linear algebra by working on these problems.

Assessment

There are two types of assessment: *summative assessment*, which evaluates your knowledge of the course, and *formative assessment*, which gives you feedback about what you need to improve. The purpose of the final exam is summative (to measure how well you know linear algebra). The purpose of the midterms and the online/written/presentation-based homework is formative (to give you feedback to improve). Unfortunately, we don't have the resources to give you feedback on every aspect of the course, so you will hold yourself accountable for working through the practice problems. It is important for you to do *all* practice problems that come at the end of each textbook module. You will not be assigned marks for the practice problems, but they are essential to your learning. To reiterate, *only working on problems that are assigned "marks" is insufficient to succeed in this class. Take learning into your own hands!*

Online Homeworks 10%

We will use the WebWork platform for online homework. Online homework will be accessible through the course webpage and must be completed by 11:59pm on the Sunday of each week. You will have five attempts at each homework problem. Your lowest four online homework scores will be dropped. Your remaining homework assignments each count for equal weight.

Perusall 10%

Before class each week, you have assigned reading. Reading math is different than reading a novel and requires constantly asking questions and developing ideas. To facilitate this kind of reading, we will use the online tool *Perusall.com*, where you can read, comment, and highlight passages. You are responsible for writing *six high-quality comments per module* (both annotations and responses/followups to others' annotations will count towards your six). Details on what "high-quality" means can be found on the course webpage.

For the first 4 Perusall assignments, your scores will be considered "practice" scores that will not count towards your course mark. This is so you can get used to the Persuall scoring. At the end of the term, if some of your "practice" scores are higher than your other scores, your lower scores will be replaced by the "practice" scores (for up to four modules).

Written Homeworks 10%

A goal of this course is to help you become proficient in expressing your mathematical ideas clearly and correctly. This takes effort and can *only* be achieved with practice and feedback. There will be 3 equally-weighted written homeworks (PAR assignments) throughout the term *which will be assigned points based on effort.* You will also receive feedback on these assignments pertaining to your mathematical writing. The purpose of these homeworks is to help you practice; it is *not* to evaluate your skill. As such, don't just try to "get the answer" when working on these assignments—try and learn to express yourself.

Most writing assignments will include three parts: (i) a PAR draft, (ii) a final submission, and (iii) a reflective component. Each part will be awarded 0 or 1 points based on effort.

Midterms 40%

Midterms are scheduled for *Friday, October 8 from 5:10–7:00pm* and *Friday, November 5 from 6:10–8:00pm.* Your lowest midterm score will account for 15% of your course mark and your highest midterm score will account for 25% of your course mark.

Final Assessment 30%

A comprehensive final assessment will take place during the final exam period in December. The date and time will be determined by FAS.

Your course mark will be computed as a weighted average of your mark in each category. (See the left column for the weights.)

Missed Assessments

Exams

If you have a legitimate academic conflict with an exam time (for example, the scheduled exam time occurs during a U of T course you are registered for) and need to register for an alternative sitting, please email `mat223@math.toronto.edu` and include (1) an explanation of why you need an alternative sitting and (2) a screenshot of your ROSI/ACORN schedule showing a legitimate conflict.

There will be no make-up exams and unexcused missed exams will be given a score of 0. This term there are no Verification of Illness forms. Students who are absent from class for any reason (e.g., COVID, cold, flu and other illness or injury, family situation) and who require consideration for missed academic work should report their absence through the online absence declaration tool and report it to their course instructor. Instructors are not allowed to seek additional information about any declared illness. `http://www.illnessverification.utoronto.ca/index.php`

If you legitimately miss a midterm exam, your final assessment score will count as your midterm score for the missed midterm.

Online Homeworks & Written Homeworks

No late homeworks will be accepted for any reason, including illness or technology malfunction. The generous drop policy was created specifically to accommodate you if you have an emergency during a homework period. Do *not* submit a self-declaration form for missed homeworks.

Email & Etiquette

We will try to respond to emails as soon as possible, but during busy times (like before an exam) it might take several days to respond. If your situation is urgent, talk to a professor after class or in office hours.

When writing an email:

- **Put MAT223 in the subject line, use your `utoronto.ca` email, and identify yourself by name and UTORid**.
- **Be specific**. We're better able to help you if you're specific about your issue and you include all necessary information. If your situation is complex, it is best to come to office hours to discuss it.
- **Check the syllabus and course webpage first**. If your question is answered on the syllabus or the course webpage, we may not respond to your email.
- **Be professional**. Please use appropriate tone and level of formality in your emails. Do not use slang or texting abbreviations. It is tradition in North America to start emails *Dear Professor* ..., and end them, *Thank you, ...*.
- **No content questions**. If you have mathematical questions, please bring your question to office hours.

Lectures & Contacts

There are several lecture sections. (R means Thursday)

Section	Time	Room	Instructor	Email	Office
LEC0201	T11–1, R11–12	SS 2135/MP 203	S. Homayouni	homayoun@math.utoronto.ca	TBA
LEC0301	T1–3, R1–2	SS 2135	J. Siefken	mat223@math.toronto.edu	TBA
LEC0401	T3–5, R3–4	PB B150	A. Jamshidpey	armin.jamshidpey@utoronto.ca	TBA
LEC0501	W9–11, F9–10	SS 2118	A. Kairshan	kairzhan@math.toronto.edu	TBA
LEC0601	W11–1, F11–12	SS 2118	S. Yoshinobu	stan@math.utoronto.ca	TBA
LEC0701	W1–3, F1–2	LM 159	R. Ronzon Lavie	ramon.ronzon@mail.utoronto.ca	TBA
LEC5101	T6–8, R6–7	MP 102	G. Papas	george.papas@mail.utoronto.ca	TBA

The course coordinator is Professor J. Siefken. Please email any administrative concerns to mat223@math.toronto.edu.

Office hours for each instructor will be posted on the course webpage.

Tutorials

You must register in a tutorial section through ROSI/ACORN by the end of the first week of classes. Tutorials will begin Monday, Sep. 27.

Tutorial rooms will change on a week-by-week basis. *Your tutorial room will be announced on Quercus.*

Attendance in tutorials is mandatory. During tutorials, you will be working on solving complex and novel problems and additionally practicing your mathematical writing. Tutorials are *not about answers* to problems. They are about *practice*. Thus, you shouldn't expect to go over every tutorial problem during a tutorial.

Academic Resources

Math Learning Centre Tutors are available to help with MAT223 in the Math Learning Centre (MLC) during regular business hours. The location of the MLC will can be found on the *Office Hours* Quercus page.

Office Hours This course has loads of office hours. Stop by! Even if you don't have a question, it can be informative to listen to other people's questions. Year after year, students report that coming to office hours regularly was the most valuable thing they did in the term.

Accessibility Needs The University of Toronto is committed to accessibility. If you require accommodations for a disability, or have any accessibility concerns about the course, the classroom or course materials, please contact Accessibility Services http://www.studentlife.utoronto.ca as soon as possible.

English Language Instruction For information on campus writing centres and writing courses, please visit http://www.writing.utoronto.ca

Other Resources Student Life Programs and Services http://www.studentlife.utoronto.ca
Academic Success Centre http://www.studentlife.utoronto.ca/asc
Health and Wellness Centre http://www.studentlife.utoronto.ca/hwc

Academic Integrity

Academic integrity is fundamental to learning and scholarship at the University of Toronto. Participating honestly, respectfully, responsibly, and fairly in this academic community ensures that the University of Toronto degree that you earn will be valued as a true indication of your individual academic achievement, and will continue to receive the respect and recognition it deserves.

Familiarize yourself with the University of Toronto's Code of Behaviour on Academic Matters https://governingcouncil.utoronto.ca/secretariat/policies/code-behaviour-academic-matters-j It is the rule book for academic behaviour at the University of Toronto, and you are expected to know the rules.

The University of Toronto treats cases of academic misconduct very seriously. All suspected cases of academic dishonesty will be investigated following the procedures outlined in the Code. The consequences for academic misconduct can be severe, including a failure in the course and a notation on your transcript. If you have any questions about what is or is not permitted in this course, please do not hesitate to contact your instructor or the course coordinator. If you have questions about appropriate research and citation methods, seek out additional information from your instructor or from other available campus resources like the University of Toronto Writing Website. If you are experiencing personal challenges that are having an impact on your academic work, please speak to your instructor or seek the advice of your college registrar.

Schedule

Below is a preliminary schedule for the course. Since classes begin on a Thursday, a "week" in this course runs from Thursday to the following Wednesday.

Make sure to read each week's module(s) *before* the week begins. Some weeks will cover multiple modules.

Week 1
September 9–15
Module 1, Appendix 1 & Questions 1–8: Set Notation, Visualizing Sets, Linear Combinations.
Online Homework: Compute linear combinations, Solve 2×2 systems, determine consistency/inconsistency.

Week 2
September 16–22
Module 2 & Questions 9–15: Restricted Linear Combinations (convex combinations, non-negative combinations, etc.), Lines, Planes.
Online Homework: Compute unions/intersections of sets, solve $m \times n$ systems with unique solutions, row reduction algorithm, determine consistency of $m \times n$ systems.

Week 3
September 23–29
Tutorials begin. Module 3, Appendix 2 & Questions 16–23: Span, Translated Spans, Linear Independence/Dependence.
Written Homework: Due September 26 at 11:59pm.
Online Homework: Write the complete solution to a system, classify sets as linearly independent/dependent, use row reduction to find maximal linearly independent subsets.

Week 4
Sept. 30–Oct. 6
Module 4 & Questions 24–29: Dot Product, Orthogonality, Normal form of lines & planes
Online Homework: Intersect lines and planes in vector form, multiply matrices.

Week 5
October 7–13
Midterm 1, Friday, October 8 from 5:10–7:00pm. **Thanksgivings Day October 12 (no classes)**.
Modules 5, 6 & Questions 30–36: Projections, Vector components, Subspace, Basis, Dimension.
Online Homework: Compute dot products, components, and projections, produce orthogonal vectors.

Week 6
October 14–20
Modules 7, 8 & Questions 37–44: Matrices, Change of Basis.
Online Homework: Compute a basis and the dimension of a span, write a vector in a different basis.

Week 7
October 21–27
Modules 9, 10 & Questions 45–52: Matrix Transformations, Linear Transformations, Composition of Transformations.
Written Homework: Due October 24 at 11:59pm.
Online Homework: Write a matrix for a linear transformation.

Week 8
Oct. 28–Nov. 3
Module 11 & Questions 53–63: Range, Nullspace, Fundamental Subspaces, Rank.
Online Homework: Find a basis for the null space, row space, and column space, compute the inverse of a matrix.

Week 9 & 10
November 4–17
Midterm 2, Friday, November 5 from 6:10–8:00pm. **Reading break November 8–12 (no classes)**.
Module 12 & Questions 64–69: Elementary Matrices, Inverses.
Online Homework: Use inverses to solve systems.

Week 11
November 18–24
Modules 13, 14, Appendix 3 & Questions 70–81: Application of Inverses (Change of Basis II), Determinants.
Online Homework: Compute 2×2 and 3×3 determinants.

Week 12
Nov. 25–Dec. 1
Module 15 & Questions 82–88: Eigenvectors/values.
Online Homework: Compute eigenvectors and eigenvalues.

Week 13
December 2–8
Module 16 & Questions 89–91: Diagonalization.
Written Homework: Due December 5 at 11:59pm.
Online Homework: Diagonalize a matrix, determine if a matrix is diagonalizable.