

MA8

Question 13: Solution

Begin by defining vectors x &amp; y containing the x, y values, respectively:

x = [8.025; 10.170; 11.202; 10.736; 9.092]

x =

```

8.0250
10.1700
11.2020
10.7360
9.0920

```

y = [8.310; 6.355; 3.212; 0.375; -2.267]

y =

```

8.3100
6.3550
3.2120
0.3750
-2.2670

```

for entering  
x, y values

Next, define vectors v1 -&gt; v6 corresponding to the columns of the 6x6 matrix (see equation (\*)):

v1 = x.\*x

v1 =

```

64.4006
103.4289
125.4848
115.2617
82.6645

```

v2 = x.\*y

v2 =

```

66.6878
64.6304
35.9808
4.0260
-20.6116

```

v3 = y.\*y

v3 =

```

69.0561
40.3860
10.3169
0.1406
5.1393

```

v4 = x

v4 =

```

8.0250
10.1700

```

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11.2020  
10.7360  
9.0920

$$v5 = y$$

$$v5 =$$

8.3100  
6.3550  
3.2120  
0.3750  
-2.2670

$$v6 = [1; 1; 1; 1; 1]$$

$$v6 =$$

1  
1  
1  
1  
1

To determine the equation of the orbit, perform a cofactor expansion along row 1 of equation (\*). This will require the evaluation of determinants  $A1 \rightarrow A6$  defined below: } ✓

$$A1 = [v2, v3, v4, v5, v6]$$

$$A1 =$$

66.6878	69.0561	8.0250	8.3100	1.0000
64.6304	40.3860	10.1700	6.3550	1.0000
35.9808	10.3169	11.2020	3.2120	1.0000
4.0260	0.1406	10.7360	0.3750	1.0000
-20.6116	5.1393	9.0920	-2.2670	1.0000

$$A2 = [v1, v3, v4, v5, v6]$$

$$A2 =$$

64.4006	69.0561	8.0250	8.3100	1.0000
103.4289	40.3860	10.1700	6.3550	1.0000
125.4848	10.3169	11.2020	3.2120	1.0000
115.2617	0.1406	10.7360	0.3750	1.0000
82.6645	5.1393	9.0920	-2.2670	1.0000

$$A3 = [v1, v2, v4, v5, v6]$$

$$A3 =$$

64.4006	66.6878	8.0250	8.3100	1.0000
103.4289	64.6304	10.1700	6.3550	1.0000
125.4848	35.9808	11.2020	3.2120	1.0000
115.2617	4.0260	10.7360	0.3750	1.0000
82.6645	-20.6116	9.0920	-2.2670	1.0000

$$A4 = [v1, v2, v3, v5, v6]$$

$$A4 =$$

64.4006	66.6878	69.0561	8.3100	1.0000
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2 marks for  
determining the  
matrices  $A1 \rightarrow A6$

103.4289	64.6304	40.3860	6.3550	1.0000
125.4848	35.9808	10.3169	3.2120	1.0000
115.2617	4.0260	0.1406	0.3750	1.0000
82.6645	-20.6116	5.1393	-2.2670	1.0000

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$$A5 = [v1, v2, v3, v4, v6]$$

$$A5 =$$

64.4006	66.6878	69.0561	8.0250	1.0000
103.4289	64.6304	40.3860	10.1700	1.0000
125.4848	35.9808	10.3169	11.2020	1.0000
115.2617	4.0260	0.1406	10.7360	1.0000
82.6645	-20.6116	5.1393	9.0920	1.0000

$$A6 = [v1, v2, v3, v4, v5]$$

$$A6 =$$

64.4006	66.6878	69.0561	8.0250	8.3100
103.4289	64.6304	40.3860	10.1700	6.3550
125.4848	35.9808	10.3169	11.2020	3.2120
115.2617	4.0260	0.1406	10.7360	0.3750
82.6645	-20.6116	5.1393	9.0920	-2.2670

The coefficients  $c1 \rightarrow c6$  of the orbit equation  $c1*x^2 + c2*xy + c3*y^2 + c4*x + c5*y + c6 = 0$  are then:

$$c1 = \det(A1)$$

$$c1 =$$

386.8024 ✓

$$c2 = -\det(A2)$$

$$c2 =$$

-102.8954 ✓

$$c3 = \det(A3)$$

$$c3 =$$

446.0293 ✓

$$c4 = -\det(A4)$$

$$c4 =$$

-2.4764e+003 ✓

$$c5 = \det(A5)$$

$$c5 =$$

-1.4280e+003 ✓

$$c6 = -\det(A6)$$

$$c6 =$$

-1.7109e+004 ✓

(10 marks)

[Entered on MA8]

