Supplementary Problems for Assignment 3, W06

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due Feb 9, at 10AM

1 Optimality

Do any of the three points

$$x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad y = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad z = \begin{pmatrix} 3 \\ 0.5 \end{pmatrix},$$

minimize Beale's function

$$f(x_1, x_2) = (1.5 - x_1(1 - x_2))^2 + (2.25 - x_1(1 - x_2^2))^2 + (2.625 - x_1(1 - x_2^3))^2$$

over $\Re^2$? Justify your answer.

2 Products and Ratios of Convex Functions

Let I be an interval and assume that the two functions f, g map I to $\Re$. Prove the following:

1. If f and g are convex, both nondecreasing (or both nonincreasing), and positive functions, on I, then the product fg is convex on I.
2. If f and g are concave, positive, with one nondecreasing and the other nonincreasing, on I, then the product fg is concave on I.
3. If f is convex, nondecreasing, and positive, and g is concave, nonincreasing and positive, on I, then the ratio f/g is convex on I.

3 Constrained Optimization

Verify whether one of the vectors $\begin{pmatrix} -\frac{3}{13} \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ .5 \\ -1 \end{pmatrix}$ is optimal for the optimization problem

$$\begin{array}{ll} \text{min} & \frac{1}{2}x^T Px + q^T x + r \\ \text{subject to} & -1 \leq x_i \leq 1, \quad i = 1, 2, 3, \end{array}$$

where

$$P = \begin{pmatrix} 13 & 12 & -2 \\ 12 & 17 & 6 \\ -2 & 6 & 12 \end{pmatrix} \quad q = \begin{pmatrix} -11 \\ -14.5 \\ 13 \end{pmatrix}, \quad r = 1.$$